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Behavior of an Interferometer in a Gravitational Field*

HERBERT E. IVES
Bell Telephone Laboratories, New York, New York
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THE special theory of relativity grew out of the need for an explanation of the results obtained with an interferometer in motion. The general theory of relativity, while an extension of the special theory, is ordinarily developed without specific reference to the theory of the interferometer. It is the purpose of this paper to apply the equivalent of the assumptions used in the general theory directly to the behavior of an interferometer in motion in a gravitational field. As will be seen, this procedure gives a direct and simple derivation of those formulae of the general theory on which its test and confirmation depend.

There is no experimental information on the behavior of interferometers in gravitational fields, consequently the subject can only be treated by making assumptions and developing their consequences.

We shall assume first that an interferometer stationary in a gravitational field assumes dimensions, and "clock" rate as though in motion. For the equivalent motion we take that of a particle falling from infinity under the influence of the field. Going at once to a spherically symmetrical field about an attracting mass M, we have for this velocity, $V^2=2GM/R$, where G is the Newtonian gravitational constant, and R is the distance of the interferometer from the center of gravity of the mass. For the length and frequency contraction factors we then have

$$(1-V^2/c^2)^{\frac{1}{2}}=(1-2GM/Rc^2)^{\frac{1}{2}},$$

where c is the velocity of light in free space.[1]

We shall assume initially that the dimensions of the interferometer are small with respect to the gravitational gradient, so that the contraction factors are sensibly constant over the region occupied by the interferometer. We shall assume the length of the stationary interferometer transverse to the field to be unaffected, while in the direction of the field it will be

$$L'=L(1-2GM/Rc^2)^{\frac{1}{2}},$$

where L is the undistorted length, or length when the interferometer is stationary in the absence of a field. The interval between clock "ticks" will be

$$\tau'=\tau/(1-2GM/Rc^2)^{\frac{1}{2}},$$

where τ is the period with the interferometer stationary in the absence of a field. For convenience we shall use γ for $1-2GM/Rc^2$ so that

$$L'=L\gamma^{\frac{1}{2}},$$
$$\tau'=\tau/\gamma^{\frac{1}{2}}.$$

* Read before the Optical Society of America at its meeting in New York City, February 24, 1939.

[1] Distances and times in this paper are measured, unless otherwise specified, by rods and clocks which are unaffected by motion (Lorentz' "stationary" rods and clocks). The light signals are transmitted through the luminiferous ether, with respect to which velocities are measured by rods and clocks of the above type.

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We assume finally that an interferometer which conforms to the above conditions, will, when set for a null adjustment in the absence of a gravitational field, maintain this adjustment when placed in a gravitational field.

Let us consider first that the interferometer is stationary in the gravitational field, with one arm in the radial direction, the other in the transverse direction. Consider a light signal reflected from the diagonal mirror in the direction transverse to the field. The time to go and return to the mirror is $2L/c_T$, where c_T is the velocity of light in the transverse direction. For the interferometer in the radial direction the time taken by the light signal to go and return is $2L\gamma^{\frac{1}{2}}/c_R$, where c_R is the velocity of light in the radial direction. Now in accordance with our assumption of invariance of the interferometer indication in a gravitational field, these times are to be equal to the interval indicated by the clock, namely $\tau/\gamma^{\frac{1}{2}}$. We thus have

$$2L/c_T=2L\gamma^{\frac{1}{2}}/c_R=\tau/\gamma^{\frac{1}{2}}=2L/c\gamma^{\frac{1}{2}}, \qquad (1)$$

from which we have at once

$$\begin{aligned} c_T&=c\gamma^{\frac{1}{2}},\\ c_R&=c\gamma. \end{aligned} \qquad (2)$$

Note that the *measured* velocity in either direction (when measuring rods and clocks are moved infinitely slowly) is c.

There are two immediate consequences from our assumptions and the above deductions. The first is that the rate of a "natural clock" placed in a gravitational field will be less than away from the field—a "red shift." The second is that a light wave front passing near a large mass will be changed in direction because of its different velocity at different distances from the massive body.

Before passing over to the case of a moving interferometer let us find a general expression which we shall need, namely that for the velocity of light in any direction. Consider a hollow reflecting sphere of radius a, surrounding the diagonal mirror of the interferometer. When in a gravitational field this sphere becomes an ellipsoid of radii a, a, b, where $b=a\gamma^{\frac{1}{2}}$. Elements of this ellipsoid are equivalent to mirrors of the interferometer in various positions. The time taken for a light signal to travel to the elements

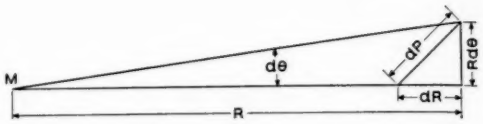

FIG. 1.

in the direction of the field, and at right angles to it are, respectively,

$$t=b/c\gamma$$

and

$$t=a/c\gamma^{\frac{1}{2}}$$

or

$$a=tc\gamma^{\frac{1}{2}},$$

$$b=tc\gamma.$$

In order to find the velocity along any radius r, at an angle θ with the direction of the field, we set down first the polar equation of the radius. Restricting our consideration to the plane at right angles to the major axes we have

$$r^2=a^2b^2/(a^2\cos^2\theta+b^2\sin^2\theta).$$

Then for the velocity c_θ of the light signal in the direction θ,

$$c_\theta^2=\frac{r^2}{t^2}=\frac{a^2b^2}{t^2(a^2\cos^2\theta+b^2\sin^2\theta)}$$

$$=\frac{c^2\gamma^2}{\cos^2\theta+\gamma\sin^2\theta}. \qquad (3)$$

We now proceed to the case of the interferometer in uniform motion with velocity v, experiencing the Fitzgerald contraction, which is proportional to $(1-v^2/c_\theta^2)^{\frac{1}{2}}$. We consider first motion in the radial direction of the field ($\theta=0$). For the time of journey of a light signal from and back to the diagonal mirror we have

$$\begin{aligned} t&=\frac{L\gamma^{\frac{1}{2}}(1-v^2/c^2\gamma^2)^{\frac{1}{2}}}{c\gamma+v}+\frac{L\gamma^{\frac{1}{2}}(1-v^2/c^2\gamma^2)^{\frac{1}{2}}}{c\gamma-v}\\ &=\frac{2L}{c\gamma^{\frac{1}{2}}(1-v^2/c^2\gamma^2)^{\frac{1}{2}}}. \end{aligned} \qquad (4)$$

For a light signal reflected from the mirror on the interferometer arm at right angles to the direction of motion we note that the actual path of the signal is around two sides of a triangle to meet the diagonal mirror moving with the velocity v. The base angle of this triangle is

$\tan^{-1} 2L/vt$. Substituting this angle in (3) we get

$$c_\theta^2=\frac{c^2\gamma^2(L^2+v^2t^2/4)}{v^2t^2/4+\gamma L^2}.$$

The path length $c_\theta t$ is $2(L^2+v^2t^2/4)^{\frac{1}{2}}$, giving the relation

$$4(L^2+v^2t^2/4)=\frac{c^2\gamma^2t^2(L^2+v^2t^2/4)}{v^2t^2/4+\gamma L^2}$$

or again

$$t=\frac{2L}{c\gamma^{\frac{1}{2}}(1-v^2/c^2\gamma^2)^{\frac{1}{2}}},$$

showing that the invariance of the interferometer behavior holds (for these representative cases) with both the gravitational and Fitzgerald contractions operative.

For motion of the interferometer transverse to the field we get by similar analysis

$$t=\frac{2L}{c\gamma^{\frac{1}{2}}(1-v^2/c^2\gamma)^{\frac{1}{2}}}, \tag{5}$$

differing from (4) only in the occurrence of γ in place of γ^2 in the contraction factor.

We have now established the method of attack sufficiently to study the most general case, that of an interferometer moving in any direction in a radial gravitational field. In Fig. 1 let the path of the interferometer be represented by $dP=(dR^2+R^2d\theta^2)^{\frac{1}{2}}$; its velocity by dP/dT. In order to derive the desired formulae we must determine the dimensions of the interferometer arms when moving in this direction, and the velocity of light in the directions taken by the signals in this interferometer.

We shall develop only the formula for light traveling along an interferometer arm parallel to the direction of motion. For the length of the interferometer arm when stationary in the gravitational field and pointed in the direction $\tan^{-1} Rd\theta/dR$, we have as before, from the polar equation of the equivalent ellipsoid,[2]

$$\begin{aligned} r^2&=L^2\cdot L^2\gamma\Big/\left(\frac{L^2dR^2}{dR^2+R^2d\theta^2}+\frac{\gamma L^2R^2d\theta^2}{dR^2+R^2d\theta^2}\right) \\ &=L^2\gamma(dR^2+R^2d\theta^2)/(dR^2+\gamma R^2d\theta^2). \end{aligned} \tag{6}$$

[2] We here assume that the ellipsoid which is equivalent to the interferometer does not change the direction of its axes. Actually the ellipse is tilted so that its major axis remains always normal to the direction of the field. This adds to $\tan^{-1} Rd\theta/dR$ a small angle, approximately $=\frac{1}{2}d\theta$, the effect of which in the formula would be only to add small times of high order.

For the velocity of light c' at the angle $\tan^{-1} Rd\theta/dR$ we have

$$c'^2/r^2=c^2\gamma/L^2$$

or

$$c'^2=\frac{r^2c^2\gamma}{L^2}=\frac{c^2\gamma^2(dR^2+R^2d\theta^2)}{dR^2+\gamma R^2d\theta^2}. \tag{7}$$

From this we have that the Fitzgerald contraction of an interferometer arm when in motion in the direction of the path is in the ratio

$$\left(1-\frac{\frac{dP^2}{dT^2}}{c'^2}\right)^{\frac{1}{2}}:1$$

$$=\left(1-\frac{dR^2/dT^2+R^2d\theta^2/dT^2}{\frac{c^2\gamma^2(dR^2+R^2d\theta^2)}{dR^2+\gamma R^2d\theta^2}}\right)^{\frac{1}{2}}:1. \tag{8}$$

Let us now send a light signal out from the diagonal mirror and back along the interferometer arm r in the direction $\tan^{-1} Rd\theta/dR$, with the apparatus moving with the velocity dP/dT. We have

$$dT=\frac{r\left(1-\frac{(dP/dT)^2}{c'^2}\right)^{\frac{1}{2}}}{c'+dP/dT}+\frac{r\left(1-\frac{(dP/dT)^2}{c'^2}\right)^{\frac{1}{2}}}{c'-dP/dT},$$

which upon making the necessary substitutions and reductions gives

$$dT=\frac{2L}{c\gamma^{\frac{1}{2}}\left(1-\frac{dR^2+\gamma R^2d\theta^2}{c^2\gamma^2dT^2}\right)^{\frac{1}{2}}}. \tag{9}$$

This formula reduces to (4) and (5) for the radial and transverse motions of the interferometer when dR and $Rd\theta$ are put equal to zero.[3]

[3] This is considered sufficient check on the general validity of the formula to permit omission of the very complicated case of light signals going around two sides of a triangle to return to the diagonal mirror, instead of along the direction of travel of the interferometer, as in this case.

Upon studying this formula we observe that a fixed factor is the undistorted length of the interferometer arm, L. If we take in place of it the diameter of the spherical shell of which the interferometer mirrors are elements, and call this $ds(=2L)$, and solve for ds^2 we get the relation

$$ds^2=c^2\gamma dT^2-R^2d\theta^2-dR^2/\gamma. \qquad (10)$$

This describes the undistorted shell diameter (twice the undistorted length of the interferometer arm) in terms of the velocity of light in free space (c), and the time (dT) taken for a light signal to traverse the interferometer arm (out and back) when the interferometer is subject to a gravitational field defined by γ, and to a Fitzgerald contraction due to its velocity in the field.

This formula (10) will be recognized as the Schwarzschild formula of general relativity, ordinarily derived by means of the tensor calculus from Einstein's three assumptions—the equivalence hypothesis, the infinitesimal hypothesis, and the general principle of relativity.

Deferring discussion of the general significance of this formula, let us continue the study of the interferometer by considering the limitation which was imposed in developing the subject, that the dimensions of the interferometer shall be so small that the gravitational field may be considered uniform over its extent. It is clear that if the interferometer is large, or traveling at high velocity, different values of γ occur at different parts of its path, and of the path of the light signal. The path of the light signal, except when exactly in the radial direction, will be curved. It appears probable, although the proof is difficult, that the integrated effect of the contractions of elementary sections of the apparatus and the curved light path will preserve for all dimensions and velocities the invariance of behavior postulated for an infinitesimal interferometer, and embodied in formula (10). If this proves to be the case, the assumed limitation to small dimensions becomes merely a device to facilitate mathematical treatment. Otherwise a sufficiently large and sensitive interferometer could be used to detect motion in a gravitational field. A different approach to the problem, which would avoid the limitation to infinitesimal dimensions just discussed would be *not* to assume invariance of readings in a gravitational field. We would then make only the single assumption of the equivalence of the effect of a stationary gravitational field to motion in a field-free space. Proceeding on this basis the velocity of light would be left constant and alike in all directions, and the conclusion would be that an interferometer of any size in a gravitational field would vary in its readings depending on its position and orientation.[4] The *measured* velocity of light would be different along and across the gravitational field. Atomic clocks in the sun would still run slow, but stars seen near the sun would not be displaced in apparent position; the latter conclusion being in conflict with observation.

The formula (10) as here derived is of purely optical significance, and must look for its support to optical experiments, such as the change of frequency of atomic light sources in massive stars, and the shift of star images near the sun, both of which are held to be observationally established. There is nothing in the derivation to suggest that the formula furnishes any information outside of optics, for instance in the field of dynamics. The formula describes (*on the basis of the assumptions made*) the behavior of light signals in an interferometer moving in a straight line in a Newtonian gravitational field. There is

[4] In this connection reference may be made to a paper by H. B. Phillips, "Note on Einstein's Theory of Gravitation" (J. Math. Phys. **1**, 177–190 (1922)) where the development is also made without the use of the tensor calculus or (explicitly) of invariance. Phillips assumes the equivalence hypothesis, but interprets its implications differently than is here done. His assumption is that "fixed" distances and times in the gravitational field will measure the same as if measured from the equivalent moving particle. The assumption in the present paper is that rods and clocks in a stationary gravitational field behave like rods and clocks on the equivalent moving particle. While superficially the same in appearance, the two assumptions are effectively different. In order to make measurements of distances and times, rods and clocks must be used, and must be transported on the measuring platforms (J. Opt. Soc. Am. **27**, 263 (1937)). Rods and clocks transported on the moving (freely falling) platform, do not yield the same measurements as do rods and clocks of identical behavior transported on the *stationary* platform in the gravitational field. Phillips' formula for ds is in terms of distances and times measured on the gravitating body, while ours is in the invariant system (Note 1). Our formula corresponding to his would be of the same form as in a field-free space (i.e., no γ's) and would embody the assumption that the *measured* velocity of light is constant (invariance of interferometer behavior). Phillips' formula for ds thus calls for variation of interferometer indications in a gravitational field.

in it no non-Euclidean geometry, nor new law of gravitation. It calls for anisotropy of the ether due to the presence of a center of gravitational attraction, but makes no suggestion as to how this anisotropy affects other than optical phenomena.

The identification of ds with the (undistorted) length of the interferometer must be pondered, in connection with the somewhat mystical character[5] which is ordinarily ascribed to this quantity in using the formula (10) to derive the motion of a particle in a gravitational field. This latter use of the formula identifies ds with the distance traveled by the particle in a four-dimensional distorted space-time of imaginary coefficients. The philosophy of this procedure lies outside the scope of the present paper. Suffice it to recall that by constraining the particle to move in this space-time along the path governed by the condition $\delta \int ds = 0$, Einstein derives the advance of the perihelion of Mercury.[6]

[5] In this connection the following statement by T. Levi-Civita (Nuovo Cimento **13**, 45 (1936)) is of interest: "During a conversation which I had with Einstein a few years ago I asked him . . . if it is possible to give any concrete interpretation . . . to the elementary chronotopic interval ds^2 . . . . He replied to me that he did not know of any physical meaning . . . ."

[6] For an alternative treatment, using Hamilton's theorem of ordinary dynamics, see H. A. Lorentz, *Problems of Modern Physics*, Note 18, p. 278.

Standardization of the Luminous Transmission Scale Used in the Specification of Railroad Signal Glasses

KASSON S. GIBSON AND GERALDINE WALKER HAUPT
Colorimetry and Spectrophotometry Section, National Bureau of Standards, Washington, D. C.

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This is the first of several papers dealing with the development and description of the signal glass specifications formulated by the Signal Section of the Association of American Railroads in 1935 and 1938. The present paper gives the spectral transmissions of the basic standards—red, yellow, green, blue, purple and lunar white glasses—on which the A. A. R. scale of luminous transmission is based, and defines that scale in fundamental, absolute units. Comparison is made with the scales defined in the 1908 and 1918 signal glass specifications.

I. INTRODUCTION

THIS is the first of several papers describing the work of the National Bureau of Standards on the color standardization of railroad signal glasses. This standardization has resulted from the cooperative efforts of the Signal Section of the Association of American Railroads (formerly the American Railway Association), Corning Glass Works, and the National Bureau of Standards. The Bureau's work was actively started in 1930 and has been formally reported to the cooperating organizations at various times since then.[1] These papers will present a summary of these reports, to which the reader is referred for additional details, illustrate and discuss the new specifications resulting from this work, *viz.*, A. A. R. Signal Section Specification 69-35, 69-38[2] and 59-38,[3] and give supplementary information not elsewhere published. Comparison will also be made with other signal glass specifications.

The authors acknowledge with pleasure the interest and support of Mr. J. C. Mock, Signal Engineer, Michigan Central Railroad, who has been identified with the railroad signal glass standardization since 1904 and who has served as chairman of subcommittee A (of committee VI, Designs, Signal Section) in charge of the standardization of pressware and disk glasses, and of Mr. A. S. Haigh, chairman of subcommittee C in charge of the standardization of lantern glasses and present chairman of Committee VI; also the invaluable advice and assistance of Dr. H. P. Gage, Chief, Optical Division, Corning Glass Works, who has prepared

[1] Standardization of railway signal glasses—Reports on measurements and investigations undertaken by the Colorimetry Section of the National Bureau of Standards at the request of the Signal Section, American Railway Association. Reports Nos. 1 to 5, K. S. Gibson and Geraldine K. Walker; published in Signal Section Proceedings, A. R. A. **30**, 384 (1933):

Report No. 1. The transmission (A. R. A. scale) of 36 specimens of signal glass relative to transmission of 6 A. R. A. standards marked "J. C. Mock 10-3-30," a report on measurements made at Corning Glass Works, December 9–11, 1930. (June 1, 1932.)

Report No. 2. Measurements of spectral and luminous transmissions leading to the derivation of new A. R. A. transmissions for the 36 glasses listed in report No. 1. (October 24, 1932.)

Report No. 3. Spectral and luminous transmissions and derivation of new values of A. R. A. transmission for the 22 "limit" glasses selected by Committee VI, A. R. A., at Corning, November 5–6, 1931 and engraved "J.C.M. 11-6-31." (December 2, 1932.)

Report No. 4. Chromaticities and luminous transmissions, with illuminants at 1900°K and 2848°K, for the 22 "limit" glasses described in Report No. 3. (January 30, 1933.)

Report No. 5. Tentative specifications for railway signal colors. (April 27, 1933.)

Reports Nos. 6 and 7, K. S. Gibson, Geraldine Walker Haupt and H. J. Keegan; published in Signal Section Proceedings, A. A. R. **36**, 136 (1939):

Report No. 6. Examination of 65 duplicate limit glasses. (July 26, 1934.)

Report No. 7. Colorimetric data leading to specification 59–38 for kerosene hand lantern globes; comparison of specifications 59–38, 69–38, and 69–35; certification of duplicate lantern glasses. (September 28, 1938.)

[2] A. A. R. Signal Section Specification 69–38, Signal Glasses (exclusive of kerosene hand lantern globes); approved, 1938. The 1935 issue of this specification appeared in that year under the number 69–35. The differences between 69–35 and 69–38 are very slight, and no change at all was made in the transmission scale. The present transmission scale will be considered to have been established in 1935, although the data on which it is based were obtained and reported in 1930–32.

[3] A. A. R. Signal Section Specification 59–38, Kerosene Hand Lantern Globes; approved, 1938. The present A. A. R. scale of transmission is incorporated in Specification 59–38 (so far as applicable) as well as in 69–38. Both specifications 59–38 and 69–38 may be obtained from Mr. R. H. C. Balliet, Secretary, A. A. R. Signal Section, 30 Vesey Street, New York, N. Y.

the large number of glasses necessary for the successful carrying out of the signal glass standardization.

A history of railroad signal glass standardization prior to 1928 has been given by Dr. Gage in an important paper entitled, "Practical Considerations in the Selection of Standards for Signal Glass in the United States."[4] In that paper are noted the signal glass specifications adopted in 1908 and 1918 by the then Railway Signal Association, the latter specification, No. 6918, being quoted in part. There are also given information on the selection and uses of signal colors and the various types of signal ware, and spectrophotometric and colorimetric data on signal glasses in use prior to the adoption of specification 69–35.[5]

II. The A. A. R. Scale of Luminous Transmission

In 1908 the Railway Signal Association, with the cooperation of Corning Glass Works, adopted certain glass roundels for use with the kerosene flame to represent the most desirable colors then available for signal purposes. The colors were designated as red, yellow, green, blue, purple and lunar white. Each of these roundels was designated arbitrarily as having a "photometric value" of 100, regardless of the absolute or true value of its luminous transmission. They were also designated as the "standard" or "medium intensity" roundels. To allow the necessary manufacturing tolerances, certain limits of photometric value above and below the standard were specified, such as "light" and "dark" limits of 120 and 80, 125 and 75, or 130 and 70, within which a glass would be acceptable for signaling purposes. Chromaticity limits were not specified as such, but the light and dark limits were apparently tacitly used as chromaticity limits.[6] By specifying the spectral transmissions

Table I. *Spectral transmissions (percent) of the standard or medium roundels, designated as having a value of* $T_{RSA} = 100$; *1908 specification.**

Wave-Length							
Fraunhofer Line	(Millimicron Equivalent)	Red	Yellow	Green	Blue	Purple	Lunar White
A	(761)	60	0	0	0	0	0
a	(716)	65	38	0	0	42	62
B	(687)	70	50	0	0	42	49
C	(656)	72	43	0	0	0	17
D	(589)	0	41	4	3	0	15
E	(527)	0	12	27	4	0	25
b	(517)	0	9	40	6	0	38
F	(486)	0	3	45	24	2	65
G	(431)	0	0	25	40	43	74
H	(397)	0	0	0	46	42	0

* Values taken from a copy of the original Specifications for Signal Roundels, Lenses and Glass Slides, Volume V, Railway Signal Association, 1908 Proceedings, kindly furnished the authors by Mr. R. H. C. Balliet, present Secretary, A. A. R. Signal Section.

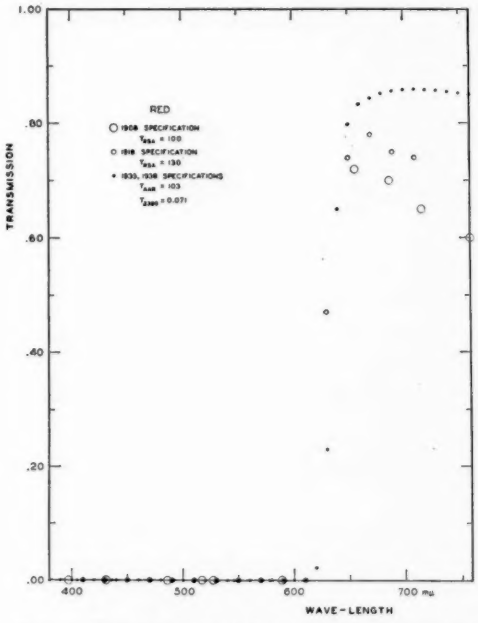

Fig. 1. Spectral transmissions defining the scales of luminous transmission for glasses designated as *red*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specifications, now in effect, are designated as A. A. R. Signal Section Specifications 59–38 and 69–38.

[4] Proceedings, International Congress on Illumination, Saranac Inn, N. Y., September, 1928, p. 834.

[5] Further information on the early history of railroad signal glass standardization is given in reports by Mr. Mock and Dr. Gage, Signal Section Proceedings, A. R. A. 30, 373, 377 (1933).

[6] However, the 1908 specification does state that "The manufacturer must submit samples of glasses showing the extreme limits of colors which it is proposed to furnish. These shall bear labels showing the photometric values . . . ." There were also given in the specification certain qualitative restrictions tending to prevent glass of undesirable color from being used. This original specification is considered further in Section IV, below.

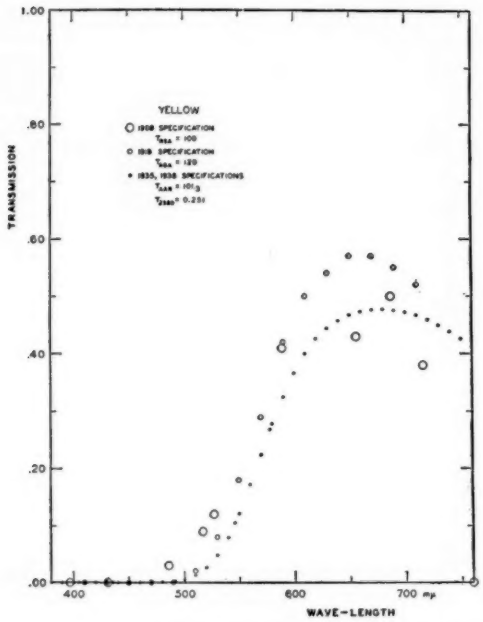

FIG. 2. Spectral transmissions defining the scales of luminous transmission for glasses designated as *yellow*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specifications, now in effect, are designated as A. A. R. Signal Section Specifications 59–38 and 69–38.

of the standard roundels, a fundamental record of the colors of the glasses was attempted. The values so specified are given in Table I and illustrated in Figs. 1 to 6.

In 1918, as a result of improvements in glass-making technique, the medium values for red, yellow and green roundels were increased from 100 to 130, 120 and 150, respectively, with corresponding increases in the light and dark limits. A new specification was formulated, designated both as 1918 and 6918, in which new tables of spectral transmission were given for the medium roundels. These values are given in Table II and illustrated in Figs. 1 to 6. They are considered further below.

This extension of the transmission (photometric) limits to higher values was based in considerable part, apparently, on values obtained by direct photometric comparisons of glasses of somewhat differing chromaticity, which introduced undesirable personal uncertainties common to heterochromatic photometry. The situation was eventually made still worse with the introduction of "electric purples," "electric lunar whites" (glasses designed for use solely with incandescent-lamp illuminants), and other types of glass differing notably in both luminous transmission and chromaticity from the original standards having transmission values designated as 100. Furthermore, the original standards had become lost. The first step in the present standardization, therefore, was to select new replicas of the lost standards and measure and define the transmissions of these new standards in fundamental terms.

Although the original standards were missing, the transmission scale itself had undoubtedly been maintained close to its original value by means of duplicate working standards at Corning Glass Works. To re-establish the scale on an official basis, Mr. Mock, in October, 1930, endorsed six roundels as the new A. R. A. standards, engraving each "J. C. Mock, 10–3–30." The designated transmissions of these standard roundels, which had been previously determined by Corning Glass Works, thereby became transmissions on the A. R. A. scale. They were as indicated in Table III. The designation, T_{ARA}, is hereinafter used to denote values of luminous transmission on this A. R. A. scale.

Although the A. R. A. Signal Section and Corning Glass Works desired that fundamental

TABLE II. *Spectral transmissions (percent) of the medium roundels, having values of* T_{RSA} *as indicated; 1918 specification as published.*

Wave-Length (Millimicrons)	Red $T_{RSA}=130$	Yellow 120	Green 150	Blue 100	Purple 100	Lunar White 100
410	0	0	40	80	90	90
430	0	0	53	73	82	80
450	0	0	62	68	74	69
470	0	0	68	54	59	65
490	0	0	70	27	18	54
510	0	2	64	10	5	38
530	0	8	48	2	1	22
550	0	18	30	2	.5	23
570	0	29	16	1	.5	26
590	0	42	6	0	0	11
610	0	50	3	0	0	12
630	47	54	1	0	0	11.5
650	74	57	0	0	0	10.5
670	78	57	0	0	.2	23
690	75	55	0	1	12	51
710	74	52	0	2	52	79

measurements be made on their standard glasses and that such standard glasses be placed in the custody of the National Bureau of Standards, there were objections to using the six standard roundels for these purposes. These roundels were unpolished and relatively large ($8\frac{3}{8}$ inches in diameter), and were therefore unsuitable for precise spectrophotometric measurements; furthermore, Corning Glass Works wished to retain them. Accordingly, Dr. Gage prepared six two-inch polished squares for each of the six signal colors, these two-inch squares duplicating the respective $8\frac{3}{8}$-inch roundels, both in chromaticity and in transmission, as closely as was feasible. At the request of the A. R. A subcommittee in charge, one of the authors thereupon went to Corning and measured the values of T_{ARA} for each of these 36 squares in terms of the respective standard roundels, using the same apparatus and method as had been used for several years in the maintenance of the standards at Corning Glass Works.[7] The values obtained are given in column 4 of Table IV.

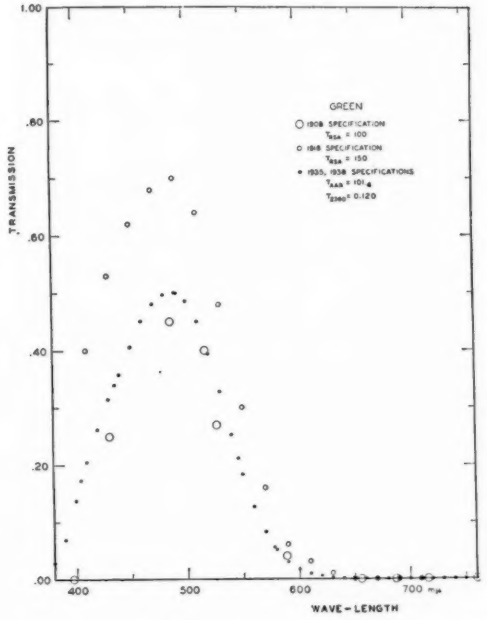

FIG. 3. Spectral transmissions defining the scales of luminous transmission for glasses designated as *green*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specifications, now in effect, are designated as A. A. R. Signal Section Specifications 59–38 and 69–38.

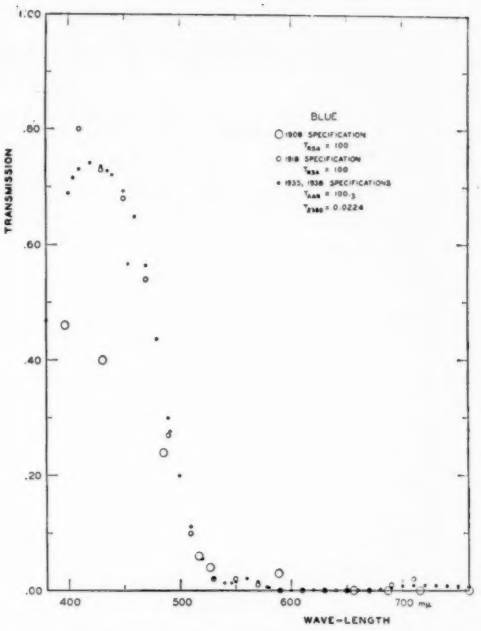

FIG. 4. Spectral transmissions defining the scales of luminous transmission for glasses designated as *blue*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specifications, now in effect, are designated as A. A. R. Signal Section Specifications 59–38 and 69–38.

It had been agreed[8] that, of these six two-inch glass squares of each color, one pair was to be deposited with the National Bureau of Standards, one pair to be deposited with the secretary or other designated custodian of the A. R. A., and one pair to remain the property of the Corning Glass Works Optical Laboratory. The Bureau's glasses are identified in Table VI as B and B', the A. R. A. glasses as A and A', and the Corning glasses as C and C'. After the measurements at Corning, noted above, these glasses were taken

[7] Details of these measurements are given in Report No. 1, Signal Section Proceedings, A. R. A. **30**, 384 (1933).

[8] Memorandum entitled, Plan for Reexamination and Duplication of A. R. A. Color Standards and Limits now Deposited with Corning Glass Works; this memorandum was prepared by Dr. H. P. Gage and represented the conclusions of the subcommittee of Committee VI, A. R. A. Signal Section, meeting at Corning on October 3, 1930.

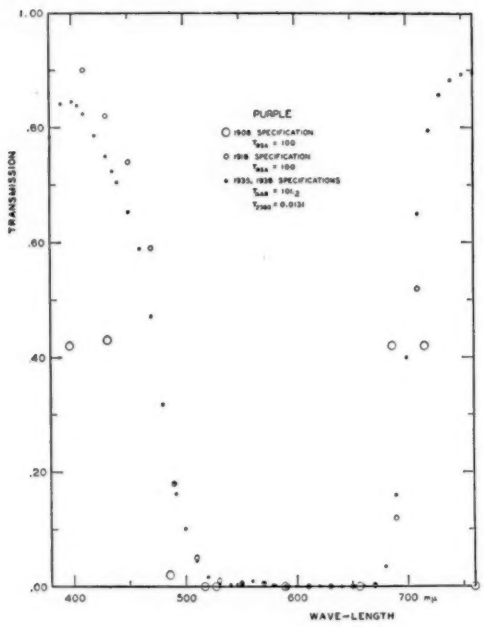

FIG. 5. Spectral transmissions defining the scales of luminous transmission for glasses designated as *purple*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specification, now in effect, is designated as A. A. R. Signal Section Specification 69–38.

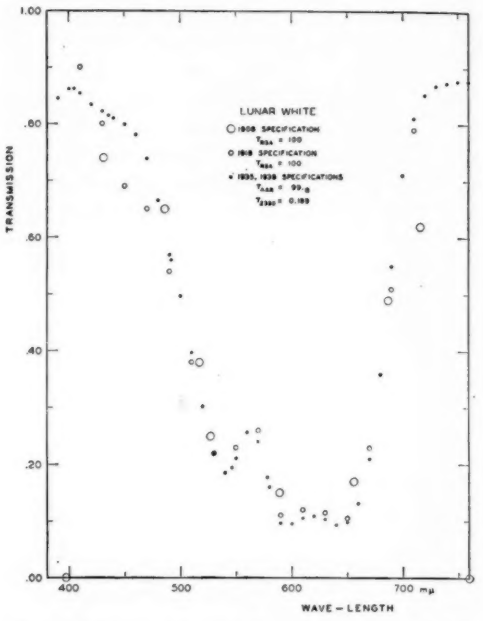

FIG. 6. Spectral transmissions defining the scales of luminous transmission for glasses designated as *lunar white*, as established, respectively, in the 1908 and 1918 specifications of the Railway Signal Association and in the 1935 and 1938 specifications of the Signal Section of the Association of American Railroads. The 1938 specification, now in effect, is designated as A. A. R. Signal Section Specification 69–38.

to the Bureau for further measurement, it having been further agreed that when values had been finally assigned these glasses by the National Bureau of Standards they would then "become the official primary A. R. A. standards and all others, including the present Corning standards, are to be subsidiary to them."

These further measurements at the Bureau are described below. On the basis of these measurements the individual values of T_{ARA} given in column 4 of Table IV were to be revised if necessary to secure more precise relative values among the six glasses of a given color; but such revision was not to change the mean values of T_{ARA} given in column 4 for each color. These mean values were to be retained as correct and final, on the basis of which a fundamental definition of the A. R. A. scale of transmission was to be adopted.

III. Expression of the A. A. R. Scale of Transmission on a Fundamental, Absolute Basis

The measurements on these glasses at the Bureau consisted of the following:

1. Measurement of the spectral transmission of the B glass of each color.—The values obtained are given in Table V and are illustrated in Figs. 1 to 6. The instruments and methods, visual and

TABLE III. *Designations of roundels selected in 1930 as the basis of the A. R. A. scale of luminous transmission.*

Identifying number of roundel	Color designation of roundel	Corning values of transmission on A. R. A. scale
5	Red	97
0	Yellow	100
4	Green	100
6	Blue	105
4	Purple	101.3
3	Lunar white	100

photoelectric, have been adequately described previously.[9] Measurements of transmission (where measurable) were made by one or both

[9] K. S. Gibson, "Direct-Reading Photoelectric Measurement of Spectral Transmission," J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 693 (1923); H. J. McNicholas, "Equipment for Routine Spectral Transmission and Reflection Measurements," Nat. Bur. Stand. J. Research **1**, 793 (1928) RP 30; K. S. Gibson, "Spectrophotometry at the Bureau of Standards," J. Opt. Soc. Am. **21**, 564 (1931).

TABLE IV. *Data obtained by one of the authors (K. S. G.) at Corning Glass Works on the glasses selected to replace the former A. R. A. standard roundels.*

1	2	3	4	5	6	7
		A. R. A. TRANSMISSION				
COLOR DESIGNATION	NUMBER ENGRAVED ON GLASS	AS MARKED ON LABEL	AS DETERMINED BY K. S. G.	COLUMN 4 MINUS COLUMN 3	COLOR RELATIVE TO STANDARD	THICKNESS (MM)
Red	83	95.2	97.5	+2.3	Red	1.60
	86	101	102.6	+1.6	Match	1.61
	88	105	104.6	−0.4	Match	1.61
	91	100	103.1	+3.1	Yellow	5.21
	92	100	106.4	+6.4	Yellow	3.84
	93	97	102.0	+5.0	Yellow	4.15
	Mean	(99.7)	102.7	+3.0		
Yellow	114	100	101.3	+1.3	Red	6.60
	115	100	101.7	+1.7	Red	6.60
	116	100	102.5	+2.5	Red	6.60
	117	100	102.1	+2.1	Red	6.60
	118	100	101.1	+1.1	Red	6.60
	138	100	101.5	+1.5	Red	6.59
	Mean	(100)	101.7	+1.7		
Green	110	102	103.1	+1.1	Match	3.68
	111	102	102.7	+0.7	Match	3.68
	112	102	102.1	+0.1	Match	3.70
	113	102	103.3	+1.3	Match	3.68
	120	100	100.6	+0.6	Match	3.74
	121	100	101.5	+1.5	Match	3.74
	Mean	(101.3)	102.2	+0.9		
Blue	52	100	101.4	+1.4	——	4.97
	53	100	100.6	+0.6	More Sat.*	4.99
	54	100	99.4	−0.6	——	5.00
	79	100	99.7	−0.3	More Sat.	5.00
	81	100	99.2	−0.8	——	5.00
	82	100	100.7	+0.7	——	4.99
	Mean	(100)	100.2	+0.2		
Purple	38	100	101.0	+1.0	Match	4.86
	39	100	101.0	+1.0	Match	4.86
	60	100	101.0	+1.0	Match	4.83
	66	101	101.2	+0.2	Match	4.83
	67	101	101.5	+0.5	Match	4.83
	68	100	100.3	+0.3	Match	4.84
	Mean	(100.3)	101.0	+0.7		
Lunar white	40	101	101.0	0.0	Blue	6.18
	49	99.5	100.0	+0.5	Blue	6.24
	67	100	98.9	−1.1	Blue	6.22
	68	100	100.1	+0.1	Blue	6.21
	69	100	99.5	−0.5	Blue	6.23
	71	100	99.9	−0.1	Blue	6.21
	Mean	(100.1)	99.9	−0.2		

* The record shows comments on but two of the six blue glasses. It is uncertain whether the other four should be designated "more saturated" or "match."

TABLE V. *Spectral transmissions of the A, B and C glasses (comprising 18 of the 36 glasses listed in Tables IV and VI.*

	Red			Yellow			Green			Blue			Purple			Lunar White		
Wave-Length (millimicrons)	92 *A*	86 *B*	83 *C*	115 *A*	118 *B*	114 *C*	111 *A*	121 *B*	110 *C*	53 *A*	82 *B*	54 *C*	38 *A*	50 *B*	66 *C*	71 *A*	68 *B*	49 *C*
Hg 404.7		0.0000			0.0000			0.173			0.716			0.838			0.862	
Hg 435.8		.0000			.0000			.340			.728			.724			.815	
Hg 491.6		.0000			.0026			.500			.277			.162			.560	
Hg 546.1		.0000			.1053			.211			.0130			.0031			.194	
Hg 578.0		<.0001			.268			.0554			.0060			.0022			.177	
380	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.028	0.028	0.028	0.477	0.468	0.472	0.783	0.783	0.783	0.795	0.795	0.795
90							.070	.069	.070	.616	.604	.610	.845	.841	.840	.845	.845	.846
400							.139	.138	.139	.695	.689	.693	.852	.845	.844	.862	.861	.862
10							.207	.205	.206	.738	.731	.737	.830	.824	.824	.854	.854	.855
20							.264	.262	.264	.749	.742	.749	.791	.786	.786	.834	.834	.834
30							.317	.315	.317	.742	.736	.742	.755	.750	.750	.822	.822	.823
40							.360	.358	.359	.726	.721	.726	.709	.705	.705	.810	.810	.812
450				.0000	.0000	.0000	.407	.406	.407	.699	.693	.698	.657	.653	.653	.799	.799	.801
60				.0001	.0001	.0001	.451	.451	.452	.655	.649	.653	.593	.589	.589	.780	.781	.783
70				.0003	.0003	.0003	.481	.481	.481	.599	.564	.567	.473	.471	.473	.738	.739	.741
80				.0008	.0008	.0008	.497	.497	.497	.440	.437	.439	.318	.318	.320	.664	.665	.667
90				.0022	.0021	.0021	.501	.501	.501	.302	.300	.301	.181	.181	.182	.569	.569	.570
500				.0056	.0054	.0055	.486	.486	.485	.201	.200	.200	.1010	.1015	.1023	.497	.497	.497
10				.0132	.0128	.0130	.450	.450	.450	.1120	.1115	.1112	.0443	.0447	.0449	.397	.397	.397
20				.0266	.0261	.0265	.395	.394	.395	.0551	.0550	.0546	.0161	.0163	.0163	.302	.302	.301
30				.0494	.0483	.0489	.330	.328	.329	.0222	.0222	.0220	.0048	.0049	.0049	.220	.220	.219
40				.0814	.0798	.0807	.255	.253	.255	.0131	.0132	.0131	.0027	.0028	.0028	.184	.185	.184
550				.1237	.1218	.1228	.185	.183	.185	.0152	.0152	.0150	.0042	.0043	.0043	.210	.211	.210
60				.174	.172	.173	.1289	.1268	.1289	.0210	.0210	.0208	.0091	.0092	.0092	.257	.257	.257
70		.0000		.226	.224	.225	.0844	.0824	.0844	.0146	.0147	.0145	.0067	.0069	.0069	.241	.241	.240
80		.0001	.0000	.280	.278	.280	.0525	.0509	.0526	.0045	.0046	.0045	.0016	.0017	.0017	.160	.160	.159
90	.0000	.0002	.0002	.327	.325	.326	.0309	.0296	.0309	.0011	.0011	.0011	.0003	.0003	.0003	.0964	.0962	.0954
600	.0001	.0005	.0004	.368	.366	.367	.0174	.0165	.0174	.0008	.0008	.0008	.0002	.0002	.0002	.0951	.0950	.0938
10	.0013	.0019	.0013	.402	.400	.401	.0099	.0093	.0099	.0008	.0008	.0008	.0003	.0003	.0003	.1059	.1058	.1047
20	.065	.022	.015	.428	.426	.428	.0055	.0051	.0055	.0006	.0006	.0006	.0004	.0004	.0004	.1090	.1090	.1082
30	.38	.23	.18	.446	.444	.446	.0030	.0027	.0030	.0004	.0004	.0004	.0003	.0003	.0003	.1036	.1035	.1025
40	.60	.65	.61	.459	.457	.458	.0017	.0015	.0017	.0003	.0003	.0003	.0002	.0002	.0002	.0936	.0935	.0923
650	.678	.798	.785	.469	.467	.468	.0009	.0008	.0009	.0002	.0002	.0002	.0003	.0003	.0003	.0980	.0980	.0970
60	.717	.833	.823	.475	.473	.474	.0005	.0004	.0005	.0004	.0004	.0004	.0008	.0008	.0008	.132	.132	.131
70	.738	.844	.834	.478	.476	.477	.0002	.0002	.0002	.0009	.0009	.0009	.005	.005	.005	.21	.21	.21
80	.754	.852	.841	.479	.477	.479	.0001	.0001	.0001	.0021	.0021	.0021	.034	.035	.036	.36	.36	.36
90	.764	.856	.846	.477	.475	.477	.0000	.0000	.0000	.0045	.0045	.0044	.16	.16	.16	.55	.55	.55
700	.769	.858	.848	.474	.472	.473				.008	.008	.008	.40	.40	.40	.71	.71	.71
10	.772	.859	.850	.468	.467	.468				.009	.009	.009	.65	.65	.65	.810	.810	.811
20	.770	.858	.850	.460	.459	.460				.0085	.0085	.0085	.798	.795	.795	.851	.852	.853
30	.768	.857	.849	.450	.449	.450				.008	.008	.008	.860	.857	.857	.868	.868	.869
40	.764	.855	.848	.439	.438	.439				.0075	.0075	.0075	.885	.883	.883	.873	.873	.873
750	.758	.852	.845	.427	.426	.427				.007	.007	.007	.896	.893	.893	.876	.877	.877
60	.751	.849	.842	.413	.412	.412	.0000	.0000	.0000	.0065	.0065	.0065	.900	.896	.896	.876	.877	.877
Luminous Transmission, T_{2320} ($C_2 = 14{,}350$)	0.07341	.07110	.06628	.2526	.2508	.2520	.1218	.1205	.1217	.02246	.02240	.02233	.01299	.01306	.01312	.1889	.1890	.1882

methods at every 10 mμ from 380 to 760 mμ, in addition to visual measurements at various wave-lengths of the Hg and He spectra from 404.7 to 667.8 mμ. The overlapping of the wave-length ranges of the visual and photoelectric data was extensive in all cases and the agreement of the data usually good. The values adopted, Table V, are considered uncertain in the last figure given.

2. Measurement of the ratios of transmission of the A and C glasses to the respective B glasses as a function of wave-length.—The values obtained are illustrated in Report No. 2.[10] From these measured ratios of transmission and the spectral transmissions of the *B* glasses, the spectral transmissions of the *A* and *C* glasses were computed. These are also given in Table V.

The uncertainty in these values for the *A* and *C* glasses is not much greater than that in the respective values for the *B* glasses, because the ratios of transmission, T_A/T_B and T_C/T_B could be determined as a function of wave-length with high precision. This was possible since for each

[10] Signal Section Proceedings, A. R. A. **30**, 390 (1933).

of the colors, except red, the *A*, *B* and *C* glasses were of the same melt and approximately of the same thickness and therefore had nearly identical spectral transmissions.

3. Derivation of the luminous transmissions of the A, B and C glasses.—Luminous transmission, T_θ, is herein defined as

$$T_\theta = \Sigma E_\theta V T / \Sigma E_\theta V,$$

in which θ is the color temperature of the illuminant, E_θ is the relative spectral energy of the illuminant, V is the relative spectral luminosity (visibility) function, T is the spectral transmission; and in which the summations were made with values taken at every 10 mμ from 400 to 760 mμ. The value of θ was taken at 2360°K,[11] this being the color temperature at which, approximately, similar determinations had for several years been made at Corning Glass Works. Values of E_{2360} were taken from a published table;[12] values of V were those adopted by the International Commission on Illumination in 1924.[13] The luminous transmissions for illuminant at 2360°K, T_{2360}, were thus computed and are given in the respective columns at the bottom of Table V.

Figure 7 has been prepared to assist in imparting an understanding of the computation of T_{2360}, as just outlined. In this figure are given the spectral distribution curves of (1) E_{2360}, arbitrarily unity at 560 mμ, (2) V, arbitrarily unity at the maximum, (3) the light, VE_{2360} (the product of V and E_{2360}, taken at each wavelength) for a source at 2360°K, and (4) the light from this source transmitted through the respective *B* glasses having spectral transmissions, T, this transmitted light having the distribution TVE_{2360}. The ratio of summations given above in defining T_{2360} is, for each of the six colors, equivalent to the ratio of the area beneath the TVE_{2360} curve to the area beneath the VE_{2360} curve. Furthermore, of course, the areas beneath the TVE_{2360} curves have the same relative values as do the respective values of T_{2360}, the area beneath the curve for the yellow glass being the greatest, that beneath the curve for the purple glass the least, just as the luminous transmission of the yellow glass is the greatest and that of the purple glass the least.

4. Determination of the relative luminous transmissions of the A, B and C glasses of each of the six colors from the computed values of T_{2360}.—These relative transmissions are given in column 5 of Table VI, that for the *B* glass being taken as unity in all cases.

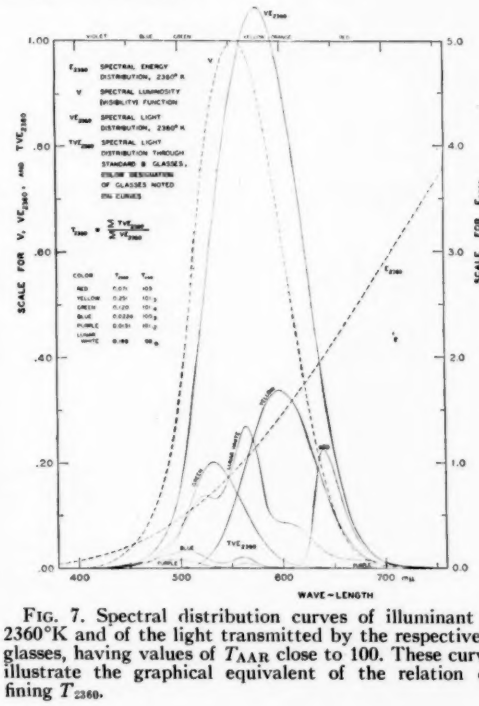

FIG. 7. Spectral distribution curves of illuminant at 2360°K and of the light transmitted by the respective *B* glasses, having values of T_{AAR} close to 100. These curves illustrate the graphical equivalent of the relation defining T_{2360}.

[11] With $C_2 = 14{,}350$ micron-degrees. For continuity with the published reports, all values of color temperature in this paper are expressed on this basis. On the color-temperature scale more recently established at the Bureau (H. T. Wensel, D. B. Judd and W. F. Roeser, Nat. Bur. Stand. J. Research **12**, 527 (1934), RP 677), for which $C_2 = 14{,}320$, the same energy distribution is given by 2355°K. For values of $C_2 = 14{,}330$ (J. F. Skogland, Nat. Bur. Stand. Misc. Pub. No. 86) and 14,360 (H. T. Wensel, "International Temperature Scale and Some Related Physical Constants," Nat. Bur. Stand. J. Research **22**, 375 (1939) RP1189), the respective values of θ are equal to 2357 and 2362.

[12] R. Davis and K. S. Gibson, "Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature," Nat. Bur. Stand. Misc. Pub. No. 114, Table 2 (1931), M 114. However, the values there given do not extend above 720 mμ. Values from 730 to 760 mμ were used as published in Report No. 2.

[13] Proceedings, Sixth Meeting, I. C. I., Geneva, p. 67. These adopted values are those recommended by Gibson and Tyndall ("Visibility of Radiant Energy," Sci. Pap. Nat. Bur. Stand. **19**, 131 (1923), Table 3, S 475) and are incorporated in the so-called 1931 I. C. I. standard observer (Proceedings, Eighth Meeting, I. C. I., Cambridge, p. 19 (1931). D. B. Judd, "The 1931 I. C. I. Standard Observer and Coordinate System for Colorimetry," J. Opt. Soc. Am. **23**, 359 (1933)).

TABLE VI. *Derivation of new A. R. A. transmissions (redefined as A. A. R. transmissions) for the 36 glasses listed in Table IV.*

DESIGNATION			RELATIVE LUMINOUS TRANSMISSION (AT 2360°K) AS DETERMINED AT NATIONAL BUREAU OF STANDARDS IN 1932					A. R. A. TRANSMISSION				
COLOR	REF.	NO.	MEAN, BY 3 OBSERVERS WITH MARTENS PHOTOMETER	AV. DEV. OF EACH OBSERVER FROM MEAN	COMPUTED FROM SPECTRAL TRANSMISSION	MEAN, COLUMNS 3 AND 5	DEV. FROM MEAN, COL. 3 (OR 5) MINUS COL. 6	AS LABELED AT CORNING	BY K. S. G. AT CORNING	NEW A. R. A. TRANSMISSION T'_{ARA}	LUMINOUS TRANSMISSION T_{2360}	T_{2360} EQUIVALENT TO $T'_{ARA} = T_{AAR} = 100.0$
Red	*C*	83	0.9383	0.0051	0.9323	0.9353	±.0030	95.2	97.5	96.2	0.06628	0.06890
	B	86	1.0000		1.0000	1.0000	.0000	101.	102.6	102.9	.07110	.06910
	B′	88	1.0303	.0021		1.0303		105.	104.6	106.0		
	C′	91	1.0022	.0213		1.0022		100.	103.1	103.1		
	A	92	1.0335	.0120	1.0325	1.0330	.0005	100.	106.4	106.3	.07341	.06906
	A′	93	.9886	.0103		.9886		97.	102.0	101.7		
	Means			.0102		.9982	(f = 102.89)		102.7	102.7		.0690
Yellow	*C*	114	1.0056	.0015	1.0047	1.0052	.0004	100.	101.3	101.8	.2520	.2475
	A	115	1.0098	.0022	1.0073	1.0086	.0012	100.	101.7	102.2	.2526	.2472
	A′	116	1.0051	.0018		1.0051		100.	102.5	101.8		
	B′	117	1.0038	.0017		1.0038		100.	102.1	101.7		
	B	118	1.0000		1.0000	1.0000	.0000	100.	101.1	101.3	.2508	.2476
	C′	138	1.0011	.0011		1.0011		100.	101.5	101.4		
	Means			.0017		1.0040	(f = 101.29)		101.7	101.7		.2474
Green	*C*	110	1.0134	.0021	1.0098	1.0116	.0018	102.	103.1	102.6	.1217	.1186
	A	111	1.0158	.0019	1.0105	1.0132	.0026	102.	102.7	102.8	.1218	.1185
	A′	112	1.0087	.0018		1.0087		102.	102.1	102.3		
	C′	113	1.0177	.0025		1.0177		102.	103.3	103.2		
	B′	120	.9952	.0014		.9952		100.	100.6	100.9		
	B	121	1.0000		1.0000	1.0000	.0000	100.	101.5	101.4	.1205	.1188
	Means			.0019		1.0077	(f = 101.42)		102.2	102.2		.1186
Blue	*A′*	52	1.0113	.0029		1.0113		100.	101.4	101.4		
	A	53	1.0024	.0020	1.0029	1.0026	.0002	100.	100.6	100.6	.02246	.02233
	C	54	.9950	.0040	.9969	.9960	.0010	100.	99.4	99.9	.02233	.02235
	C′	79	.9924	.0037		.9924		100.	99.7	99.5		
	B′	81	.9923	.0015		.9923		100.	99.2	99.5		
	B	82	1.0000		1.0000	1.0000	.0000	100.	100.7	100.3	.02240	.02233
	Means			.0028		.9991	(f = 100.29)		100.2	100.2		.02234
Purple	*A*	38	.9942	.0024	.9944	.9943	.0001	100.	101.0	100.7	.01299	.01290
	A′	39	.9931	.0032		.9931		100.	101.0	100.5		
	B	60	1.0000		1.0000	1.0000	.0000	100.	101.0	101.2	.01306	.01291
	C	66	1.0051	.0018	1.0046	1.0048	.0002	101.	101.2	101.7	.01312	.01290
	C′	67	1.0004	.0025		1.0004		101.	101.5	101.3		
	B′	68	.9937	.0023		.9937		100.	100.3	100.6		
	Means			.0024		.9977	(f = 101.23)		101.0	101.0		.01290
Lunar white	*C′*	40	1.0214	.0015		1.0214		101.	101.0	101.9		
	C	49	.9979	.0024	.9959	.9969	.0010	99.5	100.0	99.4	.1882	.1893
	A′	67	.9977	.0022		.9977		100.	98.9	99.5		
	B	68	1.0000		1.0000	1.0000	.0000	100.	100.1	99.8	.1890	.1894
	B′	69	.9946	.0006		.9946		100.	99.5	99.2		
	A	71	.9972	.0010	.9993	.9982	.0010	100.	99.9	99.6	.1889	.1897
	Means			.0015		1.0015	(f = 99.75)		99.9	99.9		.1895
1	2		3	4	5	6	7	8	9	10	11	12

5. Direct photometric measurement of the relative luminous transmissions of all six glasses of each of the six colors.—These measurements were made with the Martens photometer by each of three observers[14] and with the illuminant at 2360°K. The mean values so obtained are given in column 3 of Table VI, and in column 4 are shown the average deviations of each observer's values from the mean values of column 3.

6. Adoption of final values of relative luminous transmission for the six glasses of each of the six colors.—These values are given in column 6 of Table VI. For the *A*, *B* and *C* glasses these relative values are the means of the values obtained by spectrophotometric and photometric methods, columns 5 and 3, respectively. For the other glasses the values of column 3 are carried directly to column 6. Different methods of weighting might perhaps have been used, but the deviations (column 7) of the *A*, *B* and *C* values in columns 5 and 3 from the mean values of column 6, in no case exceeding 0.3 percent, show that any other reasonable weighting of the values would not give importantly different results.

7. Conversion of these relative luminous transmissions to the A. R. A. scale.—It will be recalled that the mean values of column 9 of Table VI (and column 4 of Table IV) were to be accepted as correct and final. Therefore, by dividing these mean values of column 9 by the respective mean values of column 6, factors are obtained, one for each color, by which the individual values of column 6 may be converted to revised A. R. A. transmissions. These factors are given in parentheses in place of the means in columns 7 and 8, and the revised transmissions on the A. R. A. scale are given in column 10. These revised transmissions were designated as T'_{ARA} throughout Reports Nos. 1 to 6. After the American Railway Association changed its name in 1933 to Association of American Railroads, it was deemed desirable to change the symbol T'_{ARA} to T_{AAR}, eliminating the use of the prime mark. The meaning of T_{AAR} is in all cases identical to that of T'_{ARA} wherever the latter is used or published.

8. Fundamental basis of the T_{AAR} scale.—The true luminous transmissions, T_{2360}, for the *A*, *B* and *C* glasses, given in Table V, are also given in column 11 of Table VI. From the values of T'_{ARA} in column 10 and these values of T_{2360} in column 11 may be derived the value of T_{2360} equivalent to $T'_{ARA}=T_{AAR}=100.0$. Such values are shown in column 12, together with the mean values thus obtained. These mean values afford a fundamental definition and conversion of values of T_{AAR}, being based upon spectrophotometric measurements, and are the values which should result if direct photometric measurements of the luminous transmissions (T_{2360}) of glasses having values of $T_{AAR}=100.0$ were made by a large number of observers under standard conditions of observation, or if the hypothetical 1931 I. C. I. standard observer could make such measurements. The mean values of column 12 are, however, subject to considerable uncertainty in the last significant figure tabulated. The values of Table VII, have therefore been adopted as expressing the fundamental relation between values of T_{AAR} and values of T_{2360}.

It may be noted that these conversion factors are valid only at 2360°K; in fact, the T_{AAR} scale, by definition, exists only at this color temperature of illuminant. The variation of absolute luminous transmission, T_θ, with color temperature, θ, of illuminant will be shown for various signal glasses in a later paper.

The question may perhaps arise as to why, in a fundamental analysis and standardization of signal glasses and colors such as the present, the A. R. A. scale of transmission was not entirely discarded in favor of the absolute scale. It is obviously just as convenient, and from the

TABLE VII. *Relation between values of T_{AAR} and of T_{2360}, which defines the luminous transmission scale established in the 1935 and 1938 A. A. R. signal glass specifications.*

COLOR DESIGNATION OF GLASS	A. A. R. TRANSMISSION, T_{AAR}	EQUIVALENT LUMINOUS TRANSMISSION, 1931 I. C. I. STANDARD OBSERVER, ILLUMINANT 2360°K, T_{2360}
Red	100.0	0.069
Yellow	100.0	.247
Green	100.0	.119
Blue	100.0	.0223
Purple	100.0	.0129
Lunar white	100.0	*.190

* The original derived value was 0.1895 (Table VI). This was at first cut back to 0.189 and was so published in Report 5 and in J. Opt. Soc. Am. **24**, 57 (1934). Later work has, however, indicated that 0.190 is a more accurate conversion factor than 0.189.

[14] H. J. Keegan assisted the authors in these observations.

scientific standpoint preferable, to insert absolute rather than relative values in a specification. Such a change was not made, however, for the following reason:

The A. R. A. scale of transmission (with perhaps some variation, see below) has been in existence for over 30 years and has been universally used by the signal engineers and glass manufacturers to designate the luminous transmission of a glass relative to its respective standard ($T_{ARA}=100$). It would be a matter of considerable inconvenience for the engineers and manufacturers to use a new scale, and continual reference to a conversion table would be necessary for a long time. Such confusion was deemed unnecessary and was avoided by the procedure outlined in this section, whereby the existing scale was kept in effect with relatively minor revisions but was placed on a consistent absolute basis by means of the conversion factors specified.

In accord with the understanding noted earlier in the paper, the A and A' glasses have been deposited with the Signal Section of the Association of American Railroads (letter of 1–5–38, Director, National Bureau of Standards to Mr. Balliet, Secretary, A. A. R. Signal Section), the C and C' glasses have been deposited with Corning Glass Works (letter of 1–5–38, Director, National Bureau of Standards to Corning Glass Works), and the B and B' glasses are in the custody of the Colorimetry and Spectophotometry Section of the National Bureau of Standards. It should be noted, however, that the present standardization of the A. A. R. scale of luminous transmission, being based upon fundamental measurements and computations, is independent of these standard glasses which were used in its determination, and if these material standards ever become lost or damaged the accuracy and usefulness of the scale, as defined in Table VII, will in no wise be affected.

IV. Comparison of the Transmission Scales as Expressed in the 1908, 1918 and 1935 (and 1938) Specifications

It is of interest to compare the present A. A. R. transmission scale, as defined for the six signal colors by the data of Table VII, with the R. S. A. scales defined by the data of Tables I and II, taken from the 1908 and 1918 specifications, respectively. From the continuity of the signal glass standardization since 1908, as outlined above, huge differences between the three scales[15] are not to be expected. On the other hand, it would be very surprising if the personal errors of heterochromatic photometry, which have been involved in the production and the maintenance of the earlier scales and to some extent in the transfer of standards illustrated in Table IV, and the lack of precision of some of the earliest spectrophotometric data, did not cause real or apparent changes of many percent in the true luminous transmission of glasses having values of T_{RSA} or T_{AAR} equal to 100, as defined in the various specifications.

Reference to the spectral transmission data of Figs. 1 to 6 is of interest in this connection. The spectral quality of the glasses would seem to have improved since 1908, particularly in the case of the yellow, blue, purple and lunar white glasses. This is indicated by the increase of transmission at the high transmissions, accompanied in many cases by a decrease of transmission at the low transmissions. On the basis of these data one would judge that glasses of purer, more saturated colors had been developed with no loss of luminous transmission; or, of greater practical importance, that signal glassware of considerably greater transmission had been developed with no loss of distinctness of the signal color. However, it is understood that no intentional changes have ever been made in the composition of the blue, purple, or lunar white glasses. The apparent differences in these graphs are probably, therefore, largely due to spectrophotometric error in the earlier data. Changes in either the type of glass or the medium value have been made at various times with the red, yellow and green glasses, but these will be considered in the later papers.

It had been hoped to make accurate comparison of the three transmission scales from these spectrophotometric data by (1) deriving the luminous transmission, T_{2360}, of the glasses from the 1908 and 1918 data, as had been done

[15] In the preceding parts of the paper the various transmission scales to which reference is made in the present section have been mostly treated as various versions of the same scale. It is more convenient in this section to consider them as separate scales.

from the 1935 data, and (2) reducing these values of T_{2360} by the proper ratio ($T_{RSA}/100$) to get the value of T_{2360} corresponding to $T_{RSA}=100$. However, the 1908 data are so meager that the uncertainties of interpolation would largely defeat the purpose. Furthermore, their reliability is open to some question, not only for the reason noted above but also in view of (1) the erratic values for the yellow glass at the longer wave-lengths and (2) the zero values of transmission for certain of the glasses at the *A* line (761 mμ), which in view of present knowledge of the spectral absorption of glasses cannot be correct if the data at the *a* and *B* lines are reasonably accurate.

The 1918 data are more extensive and reliable, however. The data are given at every 20 mμ (Table II) and values of T_{2360} computed from these data should afford an interesting comparison. To make this comparison as valid as possible, the values so computed have been compared with similar values computed from the recent spectrophotometric data (Table V) for the same wave-lengths (every 20 mμ) instead of using the values of T_{2360} given in Tables V and VI. The values for the *B* glasses were used for this purpose. Results are given in Table VIII.

It will be noted that none of the differences is as large as ten percent. The differences for the blue and purple standards[16] are subject to large uncertainty because but one significant figure is given in the 1918 spectral transmission data for these glasses (Table II) in the region of high luminosity (510 to 650 mμ). The differences for the yellow, green and lunar white standards are subject to less uncertainty from this cause. Those for the yellow and green are probably explainable by the uncertainties of heterochromatic photometry. Reference to column 4 of Table VI shows average deviations for the red glasses up to two percent; and the chromaticity differences involved in those comparisons are probably much less than those present when the changes from 100 to 120 and 150 were made for the respective yellow and green medium glasses. (It follows, of course, that the close agreement shown in Table VIII for the red standards is fortuitous.) The reason for the difference in the values for the lunar white standards is uncertain.

Inspection of Figs. 1 to 6 would indicate that the original (1908) yellow and lunar white standards would have a T_{2360} value considerably in excess of both the 1918 and the 1935 standards, but no certain differences are indicated by the data for the standards of the other colors. As already explained, it seemed inadvisable to attempt computations of T_{2360} from the 1908 data.

Question may also be raised as to the agreement of the A. R. A. scale, as exemplified by the roundels endorsed by Mr. Mock in 1930, Table III, both with the 1918 R. S. A. scale and with the 1935 A. A. R. scale now in effect. This cannot be answered certainly without making spectral transmission measurements on the roundels selected by Mr. Mock. However, if the values of T_{ARA} were assigned these roundels by the same Corning observer as assigned the values of T_{ARA} for the six squares of each color, as given in column 3 of Table IV, it may be concluded by comparing the mean values of columns 3 and 4 of Table IV that the A. R. A. and A. A. R. scales are in close agreement. The differences are approximately 3 percent for the red, 2 percent for the yellow and 1 percent or less for the glasses of the other colors. For the green, blue, purple, and lunar white glasses, therefore, the 1930 A. R. A. scale is in very close agreement with the 1935 A. A. R. scale, and hence differs from the 1918 R. S. A. scale by approximately the same values as are given in the last column of

TABLE VIII. *Comparison of values of T_{2360} computed for values of $T_{RSA}=100.0$ from the 1918 specification, with similar values for $T_{AAR}=100.0$ computed from data on which the 1935 specification is based.*

Color Designation	T_{2360}		ΔT_{2360} (column 2 minus column 3)	Percent change in T_{2360} (column 4 divided by column 3, ×100)
	$T_{RSA}=100.0$ (1918)	$T_{AAR}=100.0$ (1935)		
Red	0.0662	0.0672	−0.0010	−1.5
Yellow	.268	.247	+ .021	+8.5
Green	.129	.119	+ .010	+8.4
Blue	.0203	.0221	− .0018	−8.1
Purple	.0134	.0124	+ .0010	+8.1
Lunar white	.199	.189	+ .010	+5.3

[16] That is, hypothetical glasses having values of T_{RSA} or $T_{AAR}=100.0$.

Table VIII. For the yellow glasses the A. R. A. scale is intermediate between the R. S. A. and A. A. R. scales. For the red glasses the A. A. R. scale appears intermediate between the other two but the various data are too uncertain for any definite statement.

In conclusion the authors wish to call attention to the pioneering work of the railroad signal engineers in color specification. No search of the literature has been made but so far as the authors are aware the 1908 specification was the first effort, at least in this country, to place the colorimetric part of a purchase specification on a fundamental basis. This specification was the result of cooperation between the Railway Signal Association and Corning Glass Works. It followed logically the very excellent treatment given in a paper[17] entitled, "The Roundel Problem," by Dr. William Churchill of Corning Glass Works.

This 1908 specification contained the following features essential to all complete and adequate colorimetric specifications:

1. It was based fundamentally on spectrophotometric analysis of the standards.

2. It specified tolerances within which the manufactured product must come.

That the spectrophotometric analyses and the colorimetric specifications were somewhat inadequate, judged by present-day criteria, in no wise detracts from the foresight and judgment leading to the formulation of a specification so far in advance of its time. The lack of spectrophotometric precision was largely remedied in the 1918 specification, but the adequate colorimetric specification of the tolerances had to wait until first the Optical Society of America[18] and then the International Commission on Illumination[19] had set up computational procedures and data suitable for such purpose.[20] It is interesting to note, however, that the "mixture diagram," so essential in specifying the chromatic properties of signal lights, was illustrated in colors and used by Dr. Churchill in his study of the characteristics of signal glassware, to which reference has just been made.

[17] Presented at the Ninth Annual Meeting of the Railway Signal Association, Niagara Falls, New York, October 10–12, 1905.

[18] Report of Committee on Colorimetry for 1920–21, L. T. Troland, Chairman, J. Opt. Soc. Am. and Rev. Sci. Inst. **6**, 527 (1922).

[19] Proceedings of the 8th Session, Cambridge, p. 19 (1931).

[20] This will be treated in the next paper.

Improved Methods of Illumination for the Measurement of Accidental Double Refraction

YNGVE BJÖRNSTÅHL
Laboratory of Physical Chemistry, University of Upsala, Upsala, Sweden
(Received February 24, 1939)

A GREAT number of measurements have been made of the so-called accidental double refraction induced in otherwise isotropic materials by magnetic, electrical and mechanical influences. The resulting effects are generally very small. Therefore half-shade methods are indispensable for the measurements. It might be supposed that the methods of illumination, which have been used in a considerable number of determinations, ought to be perfect and that the methods of measurement could be improved no more. In the following I shall show that the illumination systems in many cases could have been improved and at the same time I shall mention some more suitable arrangements and some of the precautions, that must be observed in the measurements of the small effects associated with accidental double refraction.

The optical arrangements, which have been used for magnetic, electric and mechanical double refraction show a certain degree of resemblance.

A light source Z_0 (Fig. 1), usually the exit slit of a monochromatic illuminator, is imaged by means of an illumination lens L, for instance in infinity. The light passes through the polarizing prism N and the cell C, containing the double refracting medium. R_1 and R_2 are two circular diaphragms at the ends of the cell. The light emerging from C is elliptically polarized. We want to determine the position of the axis of the artificial crystal, the ellipticity and the orientation of the major axis of the elliptical vibration. For the determination of these constants compensators or other devices (Fig. 1, K) and a half-shade (H) are used. The half-shade is observed by means of some kind of telescopic system. The sensibility of the arrangement increases with the illumination on the retina, since only small effects are considered and the level of illumination is low.

Everyone who has studied methods of determining small double refraction, knows how difficult it is to avoid double refraction in coverglasses even if these are thin or made of special annealed glass. The use of lenses between the polarizing prisms, for instance as in microscopes, is out of the question and should be avoided in the measurement of small effects. I therefore refrain from discussing illuminating devices of such a type.

The primary light source must possess an intrinsic brightness as high as possible. It is necessary to use a high intensity arc lamp of the carbon or the mercury type. It often happens at high current densities that the lateral gradient of the brightness is considerable. Furthermore this gradient is not constant but changes with time. The light source, e.g., the crater in the arc of the special type of carbon, which must be used consists of a line spectrum, superimposed upon a continuous spectrum. Besides, the carbons never are homogeneous. Photometric measurements show that the variations of brightness may be rather considerable, even within a small part of the light source. Attention must therefore be paid to the fact that the slit Z_0 is not evenly illuminated.

It must be admitted that by using an extra condenser and a slit the illumination of the different parts of Z_0 may be made constant. Such an arrangement always means a loss of light amounting up to 50 percent. Because of this fact we do not deal with this arrangement here, as there are other means at hand.

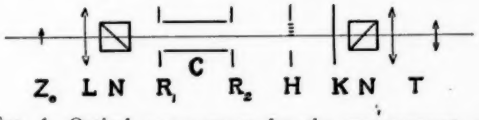

FIG. 1. Optical arrangement for the measurement of accidental double refraction.

There are two features, which, according to my opinion, must be strictly observed in all measurements of small effects. Every point of the half-shade must receive light from all parts of the light source Z_0 in order that it may be

evenly illuminated. Further, reflections must not occur. Not only the directly reflected light, which enters the system, is objectionable, but also—especially with colloids and high molecular weight substances—the diffused light of the Tyndall type, which may arise. In the following I shall indicate the possibilities existing of eliminating the two sources of disturbance, mentioned above.

A faulty arrangement is indicated by great zero-point variations in the measurements. In fact, the constancy of the zero-point position of the compensator determines the possibility of measuring small effects.

Further discussion of the matter requires a short survey of the illumination devices and their relations to the observation system and the eye. Cf. for instance, Drude.[1]

Suppose $\bar{H}$ to be the surface of the diaphragm of the half-shade; $\bar{P}$ to be the surface of the diaphragm which limits the light beam through one point of the half-shade; $\bar{H}'$ to be the surface of the image $\bar{H}$, produced by the observation system (telescope); $\bar{P}'$ to be the surface of the image of $\bar{P}$; $\bar{P}'$ is the exit pupil; $\bar{H}'$ to be the surface of the image of the half-shade on the retina; $\bar{P}''$ to be the surface of the pupil of the eye; and suppose the distance between the respective diaphragms to be designated by the following scheme (a'' meaning a constant characteristic of the eye)

$$\bar{H} \quad \bar{P} \qquad \bar{H}' \quad \bar{P}' \qquad \bar{H}'' \quad \bar{P}''$$
$$a \qquad\qquad a' \qquad\qquad a''$$

If the observation system is arranged so that the plane of the exit pupil coincides with the pupil of the eye; if $\bar{P}' \leqq \bar{P}''$ and if the brightness of the light source is i_0, the flux of light L which can be passed through the arrangement approximately is—ignoring reflection losses—

$$L = i_0 \frac{\bar{H}\bar{P}}{a^2} = i_0 \frac{\bar{H}'\bar{P}'}{a'^2} = i_0 \frac{\bar{H}''\bar{P}'}{a''^2}. \tag{1}$$

If β is the angle at which the image of the half-shade is observed by the eye and if $\tilde{\beta}$ is the corresponding solid angle,

$$\bar{H}'' = \tilde{\beta} a''^2. \tag{2}$$

[1] P. Drude, *Lehrbuch der Optik*, third edition, p. 80.

For the illumination of the image on the retina we get

$$B = L/\bar{H}'' = L/\tilde{\beta} a''^2. \tag{3}$$

The illumination is determined by the size of the image $\bar{P}'$. We must attempt to make the light flux as great and β as small as possible in order to obtain a strong illumination on the image on the retina. On account of physiological causes β cannot possibly be made smaller than $\frac{1}{2}°$. The absolute diameter of the half-shade may be of any size; by choosing a suitable telescopic arrangement the angle of observation β may always be brought to assume the optimal value.

We want to find an expression for the maximal light flux which with different experimental arrangements may be forced through the cell C (Fig. 1) with the diaphragms R_1 and R_2 and the half-shade H. We thereby presume that no part of the light beam is intercepted after its passage through H.

Arrangements for Mechanical and Electric Double Refraction

For electric and mechanical double refraction in liquids the parts (the surfaces) limiting the rays are to some extent similar. In the first case they are represented by the condenser-plates of the cell, in the second case by a limited part of two concentric cylinder surfaces. The two cases may, therefore, advantageously be treated at the same time.

Method I

If a light source (Fig. 2) is reproduced in the plane Z by means of the lens L and the extreme rays are limited by the cell diaphragm R_1 and by the half-shade diaphragm H, the half-shade will not be evenly illuminated. Suppose that R_1 is the diaphragm which limits the beam passing through any point on the half-shade. This implies $H \leqq R_2 = R_1$ where H, R_1 and R_2 represent the diameters. If we designate the distance between R_1 and H with b, we get for the light flux through a surface element $(d\bar{H})$ at the center of the half shade, the expression

$$dL = i_0 (d\bar{H}) \tilde{\omega} = i_0 (d\bar{H}) \bar{R}_1 / b^2, \tag{4}$$

where i_0 represents the intrinsic brightness for a

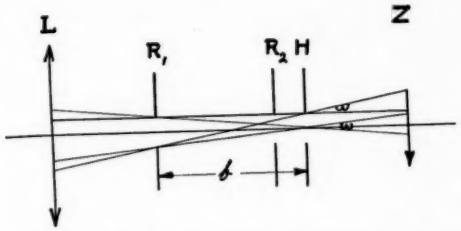

FIG. 2. Method I, inhomogeneously illuminated half-shade.

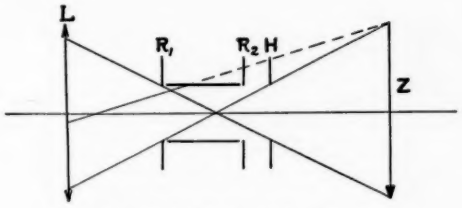

FIG. 3. Method I, reflections.

part of the light source and $\bar{R}_1$ the surface corresponding to the diameter R_1, and $\bar{\omega}$ represents the solid angle. We presume $\bar{\omega}$ to be small. For a surface element at the edge of H we obtain

$$dL_1 = i_1(d\bar{H})\bar{\omega} = i_1(d\bar{H})\bar{R}_1/b^2, \qquad (5)$$

where i_1 means the brightness of the respective part of the light source. The different light beams come from different parts of the light source with different brightness, corresponding to different parts of the image Z in the figure. The expressions are still valid whether the image Z of the light source coincides with the plane of the half-shade or is situated at infinity. Both arrangements have been used in the measurement of electric double refraction. Leiser[2] as well as several other scientists, e.g. Otterbein, Borchert[3] have used the first method, while Szivessy[4] used the latter with "parallel light."*

As regards mechanical double refraction no attention was earlier paid to the illumination devices. Vorländer and Walter[5] did not use any illumination lens. Sadran[6] made use of "parallel light."

We shall consider the conditions determining the reflections in the general case, when the image Z of the light source is located after the half-shade diaphragm. If we examine Fig. 3, we find that the reflections from the walls of the cell are produced as soon as Z is great, i.e., the condition for the nonappearance of reflections is $Z < R_1$. It is impossible to diminish the reflections by introducing a diaphragm, for instance, between the lens and R_1. An increase in the dimensions of the cell corresponding to an increase in the diaphragms R_1 and R_2 might appear of advantage. These dimensions, however, are fixed, their size depending on the available voltage and the amount of substance for electrical double refraction, and on the velocity gradient and the amount of substance available for mechanical double refraction. Since Z increases with the distance from H, the disturbances due to reflections are especially marked when Z is situated in infinity, i.e., with parallel light.† Summarizing it may be said that this method must be looked upon as of doubtful value from both theoretical and experimental point of view.

Assuming the brightness to be the same in all parts of the light source

$$L = i_0\bar{H}\bar{R}_1/b^2. \qquad (6)$$

As to the size of the half-shade diaphragm we have $\bar{H} \leqq \bar{R}_1$; thus in the most favorable case

$$L_I = i_0\bar{R}_1^2/b^2. \qquad (7)$$

Method II

If the principle of an even illumination of the half-shade is to be maintained, the conditions become more complicated, i.e., certain restrictions as regards the diaphragms are necessary. If Fig. 4 is considered, the image Z of the light source is found to be located after the half-shade H. Z limits the beam through one point of H. In this way each element of H receives light from the whole light source. If the diameters H and Z are given, the cell diaphragms R_1 and R_2 must not intercept any rays. Alternatively,

[2] R. Leiser, Diss. (Halle, 1910).
[3] G. Otterbein, Physik. Zeits. **35**, 256 (1934); L. H. Borchert, Physik. Zeits. **39**, 156 (1938).
[4] G. Szivessy, Zeits. f. physik. Chemie **26**, 326 (1924).
* The expression "parallel light" is used for convenience and does not imply that all the rays passing through the cell are strictly parallel but only that the light source is located at the focus of the illumination lens.
[5] W. Vorländer and R. Walter, Zeits. f. physik. Chemie **118**, 1 (1925).
[6] Ch. Sadran, J. de phys. et rad. **7**, 263 (1936).

† It should not be necessary to point out that no real image of Z is formed; the light beam is before that intercepted by the objective of the telescope.

we may assume R_1 and Z to be given, whereupon the size of H must not be greater than necessary to allow every ray through H and Z to pass through R_1. Let b be the distance between R_1 and H, and c the distance between H and Z, the condition for homogeneous illumination is easily obtained.

$$H \leqq (cR_1 - bZ)/(b+c). \tag{8}$$

The half-shade cannot be placed at any point after the diaphragm R_2; in any case

$$b \ll cR_1/Z. \tag{9}$$

The light flux passing through the cell, the half-shade and Z is

$$L = i_0(\pi^2/16)(H^2Z^2/c^2); \tag{10}$$

or by substitution of (8)

$$L = i_0 \frac{\pi^2}{16} \frac{Z^2}{(b+c)^2} \frac{(cR_1 - bZ)^2}{c^2}. \tag{11}$$

Supposing R_1, b and c to be constants, L has a maximum for

$$Z = cR_1/2b; \qquad H = cR_1/2(b+c) \tag{12}$$

$$L_{II} = i_0 \frac{\bar{R}_1^2}{16b^2} \left[\frac{c}{c+b}\right]^2. \tag{13}$$

If c is allowed to vary we find that L_{II}

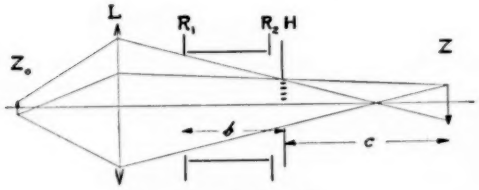

FIG. 4. Method II.

reaches its maximum value L'_{II} when the light source is imaged at infinity, i.e., when $c = \infty$

$$L'_{II} = i_0\bar{R}_1^2/16b^2. \tag{14}$$

To use parallel light, however, causes considerable reflections. See Fig. 5.

It may be of interest to examine the conditions for the occurrence of reflections with this experimental arrangement. As in case I, reflections are found to appear as soon as the image Z of the light source is great. The conditions for the non-appearance of reflections is $Z < R_1$; according to the expression (12) at maximal light flux we have

$$Z = cR_1/2b, \tag{15}$$

thus $\qquad cR_1/2b < R_1 \quad$ or $\quad c \leqq 2b.$

The maximum light flux to be used is

$$L''_{II} = i_0\bar{R}_1^2/36b^2. \tag{16}$$

From expression (12) it is evident that in any case H is smaller than R_1. The emerging light

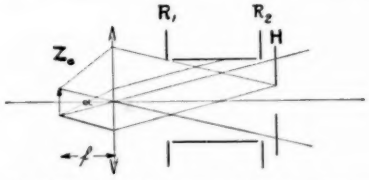

FIG. 5. Method II with parallel light, homogenously illuminated half-shade, strong reflections.

flux is small compared to that of method I. We obtain rational illumination by sacrificing the light flux or the brightness of the image on the retina. Expression (16) indicates that the half-shade should be placed as close to R_2 as possible, that is to say, near the cell. The method has been used by the author in the measurement of electric double refraction.[7]

Method III

Let us consider other possibilities. A survey of the different alternatives of placing the image Z of the light source is given in Fig. 6. The image Z may be placed at any point within the shaded areas. A natural location is in the vicinity of the diaphragm R_1. We then get an arrangement according to Fig. 7. The half-shade H may be located at any point after the cell diaphragm R_2. If the diameter H is smaller than R_2 the half-shade will be evenly illuminated; each point of it receives light from the whole image Z, which is supposed to cover the whole diaphragm R_1.

For the light flux through the arrangement

$$L_{III} = i_0\bar{R}_1\bar{H}/b^2, \tag{17}$$

where b represents the distance between R_1 and the half-shade H. It is possible to make the half-shade diaphragm as large as R_1.

$$L_{III} = i_0\bar{R}_1^2/b^2. \tag{18}$$

[7] Y. Björnståhl, Phil. Mag. **2**, 701 (1926).

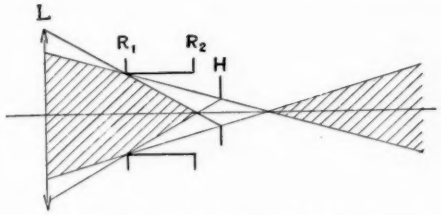

FIG. 6. Illustrating the location of the half-shade.

Comparing this expression with L'_{II} we find the last described method of obtaining even illumination superior by far as regards the light flux. As regards reflections, Fig. 7 shows that extremely marked reflections appear; the method is therefore of no value. It is evident that a diaphragm somewhere between the illumination lens and the half-shade is ineffective, it is true that it may to some extent weaken the reflections, but the light flux and the brightness of the image on the retina are reduced at the same time. In the following way it is, however, possible to arrange an illumination that is free from objectionable features. In Fig. 8, Z_0 is the light source, L the illumination lens, R_1 the first cell diaphragm and H the half-shade diaphragm. Between Z_0 and the illumination lens there is a plane H_0 corresponding to the plane in which the half-shade is located. In this plane a diaphragm is fixed, the image of which exactly covers the half-shade diaphragm H. In this way all the rays which otherwise would strike the walls of the cell and cause objectionable reflections, are intercepted without impending the rays which pass through R_1 and H. For the light flux the same expression as before is obtained.

Returning to Fig. 6, we find that it is also possible to locate Z within the cell, although in that case Z must be made smaller if an even illumination of the half-shade is to be produced. It is possible to deduce the expression for the light flux (see Fig. 9)

$$L=i_0\frac{2}{16}\frac{H^2}{b^2}\left[\frac{R_1(b+x)-Hx}{b-x}\right]^2; \qquad (19)$$

x is the distance between Z and R_1, $x<b$. The expression has a maximum for $x=0$, that is, when the image of the light source coincides with the diaphragm R_1.

The method represented by Fig. 8 possesses all the qualities of a satisfactory illumination device as specified above. The method has been used by Dr. Snellmann and the author in measurements on electric and mechanical double refraction. The superiority of this method is most obvious with substances where diffusion of light occurs.

ARRANGEMENT FOR MAGNETIC DOUBLE REFRACTION

In the case of magnetic double refraction the conditions are more complicated. Hitherto it has not been necessary to pay any attention to the divergence of the rays; the length of the cell is comparatively great, and the optical path for oblique rim rays is enlarged in the proportion $1/\cos(\alpha/2)$, where α is the angle between the oblique extreme rays. As α is small, the corresponding correction can be disregarded. In a magnetic field, however, an influence of the Faraday effect on rays forming an angle different from 90° with the field is superimposed. The plane of polarization of the extreme rays will be rotated an angle proportional to $\cos(\alpha/2)$. In order to avoid this influence α must be kept very small. The results therefore become somewhat modified. If the different methods are to be compared the angle α between the extreme rays must be kept constant.

If for magnetic double refraction H is the induction and l a coordinate at right angles to

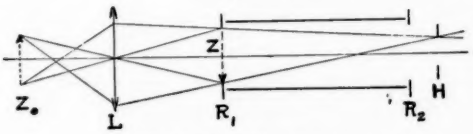

FIG. 7. Method III, reflections.

the field, then the expression $\int H^2 dl$ as a rule determines the effect. The expression reaches its highest value for wedge-shaped pole pieces of the greatest available length, that is, a length corresponding to the core of the electromagnet and increases when the air gap is reduced. On the other hand, the light flux will decrease with the air gap, which determines the diameter of the diaphragms at the end of the cell. Thus an air gap corresponding to a maximum of sensitivity

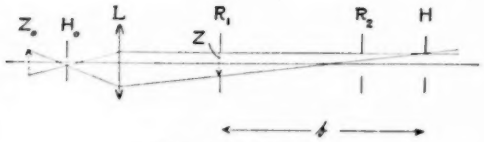

FIG. 8. Method III.

exists. Furthermore the most favorable arrangement of the pole shoes is a function of the amount of substance available and of the light absorption.

In order to give an idea of the dimensions of the cell in some preliminary experiments, carried out by means of the large electromagnet of the laboratory the following data may be of interest. The length of the pole shoe of the electromagnet was 58 cm, and that of the cell about 70 cm. The diameter of the diaphragms at the end of the cell was about 0.8 cm.

Method I_m

The method I with parallel light as outlined above has been used in the investigations made by the Cotton school;[8] the half-shade is located at a distance of about two meters from the center of the pole shoes. Parallel light has also been used by Szivessy[9] and by Boarse.[10] Difficulties appeared in the attempts to diminish the influence of reflections. Szivessy covered the inside of the cell tube with a layer of carbon, a perhaps somewhat questionable method; Boarse made the tube of copper, treated with hydrogen sulphide, in order to produce a light absorbing surface.

Assuming the brightness i_0 of the light source to be the same at all points in order to obtain simple expressions, according to the expression (6) we have

$$L=i_0\bar{H}\bar{R}_1/b^2; \quad \bar{H}=H^2\pi/4; \quad \bar{R}_1=R_1^2\pi/4; \qquad (20)$$

$$L=i_0(\pi^2/16)(H^2R_1^2/b^2). \qquad (21)$$

They can be used even in the case of parallel light. Further $H\leqq R_1$. For extreme rays, the angle between which is α, we have

$$b=(R_1+H)/\alpha.$$

In the most favorable case to be imagined*

$$L_I=i_0(\pi^2/16)(R_1^2\alpha^2/4). \qquad (22)$$

Of course the remarks on p. 204 must be applied here too: this method is not rational.

Method II_m

In order to eliminate the inefficiencies which are connected with the above described method, it is possible to modify method II. In Fig. 4, R_1 and R_2 represent the cell diaphragms, H the half-shade and Z the image of the light source. If the respective symbols correspond to the diameters of the diaphragms, we get

$$\alpha=(Z+H)/b, \qquad (23)$$

where α is the angle between extreme rays. The condition for homogenous illumination of the half-shade may be written

$$H\leqq R_1-b\alpha. \qquad (24)$$

The condition for absence of reflections is $Z\leqq R_1$. The light flux through the arrangement is

$$L_{II}=i_0\frac{\pi^2}{16}\frac{H^2Z^2}{c^2}=\frac{\pi^2}{16}\frac{(R_1-b\alpha)^2R_1^2\alpha^2}{(2R_1-b\alpha)^2}. \qquad (25)$$

If we do not pay any attention to the reflections, that is, to the condition $Z<R_1$, we get

$$L'=i_0\frac{\pi^2}{16}(R_1-b\alpha)^2\left[\frac{\alpha(b+c)-R_1}{c}\right]^2 \quad \text{and} \qquad (26)$$

L reaches its highest value for $c=\infty$

$$L'_{II}=i_0(\pi^2/16)(R_1-b\alpha)^2\alpha^2. \qquad (27)$$

It is evident that b must be made as small as possible. The half-shade should be located close

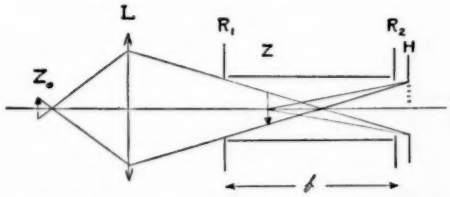

FIG. 9. Method III, location of the half-shade in the cell.

[8] M. Scherer, Thèses (Paris, 1934), p. 17.
[9] G. Szivessy, Zeits. f. Physik **18**, 97 (1923).
[10] H. A. Boarse, Phys. Rev. **46**, 187 (1936).

* It must be remembered that for various reasons it is not always possible to make an experimental arrangement corresponding to "the most favorable case."

to the second cell diaphragm R_2. This is inconvenient because of the circumstance that the half-shade ought to be located in the vicinity of the compensator, the analyzing prism etc. These should be located at some distance from the magnetic field. The method has earlier been used by the author.[11, 12]

If a comparison is made with method I_m, the same relation, which has been mentioned on p. 205 exists: method II_m gives a smaller light flux.

Method III_m

If we consider method III with the image of the light source on the first cell diaphragm R_1 (Fig. 10), we have for the light flux the expression

$$L=i_0(\pi^2/16)(Z^2H^2/b^2); \quad \alpha=(Z+H)/b. \quad (28)$$

If Z covers the first cell diaphragm

$$Z=R_1; \quad L=i_0(\pi^2/16)R_1^2(\alpha-R_1/b)^2. \quad (29)$$

In order to obtain an even illumination of H,

$$H\leqq R_1; \quad b\leqq 2R_1/\alpha. \quad (30)$$

That is to say, in the most favorable case

$$L_{III}=i_0\frac{\pi^2}{16}\frac{R_1^2}{4}\alpha^2. \quad (31)$$

The reflections may be eliminated in the manner indicated earlier by introducing a diaphragm H_0. If the light fluxes obtained by means of the different methods are compared we find

$$\frac{L_{III}}{L_I}=1; \quad \frac{L_{III}}{L_{II}}=\frac{1}{4}\left[\frac{2R_1-b\alpha}{R_1-b\alpha}\right]^2. \quad (32)$$

In an actual case,

$$\alpha=\frac{1}{200}; \quad b=80\,\text{cm}; \quad R_1=0.8\,\text{cm}; \quad \frac{L_{III}}{L_{II}}=\frac{9}{4}.$$

The last method, which locates the image of the light source at the first cell diaphragm, is, therefore, superior to all others, producing a light flux equal to that of the defective method I_m, but giving at the same time a correct illumination.

[11] Y. Björnståhl, Phil. Mag. **42**, 352 (1921).
[12] Y. Björnståhl, *Experimental Studies on the Accidental Double Refraction in Colloids* (Upsala, 1924).

If we compare L_{III} with the light flux of method II_m using parallel light, we have

$$L_{III}/L'_{II}=R_1^2/4(R_1-b\alpha)^2. \quad (33)$$

Also in this case $L_{III}>L'_{II}$, if $2b\alpha>R_1$.

A considerable advantage of this third method is also that the half-shade may be located at a suitable distance from the second cell diaphragm, that is, outside the magnetic field and close to the compensator, analyzing prism etc.

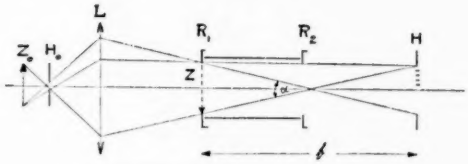

FIG. 10. Method III for a magnetic field.

The influence on the measurements of ellipticity, which results from not using strictly parallel light through the specimen cell is the same at the different methods of illumination, for a given angle α (see Fig. 10 and Fig. 5) between oblique extreme rays. Even the use of so-called parallel light (cf. footnote on page 203) does not eliminate this influence, which can be estimated with the aid of the theory for the isochromatic surfaces. Since the value of α is supposed to be small this influence can be neglected.

By using the methods of illumination described above and a compensator for instance of Brace's type it is now possible to carry out measurements on substances which could hardly be studied without these arrangements. The phenomenon is especially striking with substances with high molecular weight and with colloidal solutions, in which diffusion of the light is appreciable.

SUMMARY

It is shown that nearly all illumination devices hitherto used in the measurement of magnetic, electric and mechanical double refraction are inefficient. A description is given of methods for producing a rational illumination by means of which it is possible to carry out measurements on substances that otherwise could not be investigated.

Color Characteristics of Artists' Pigments

NORMAN F. BARNES
Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received March 13, 1939)

AT the instigation of Dr. Herbert E. Ives, an investigation of the color of artists' pigments was undertaken with the cooperation of the Department of Conservation and Research of the Fogg Art Museum, Harvard University. A brief synopsis of the results of that investigation are presented in this paper for the fifty-one pigment samples analyzed.

The method of preparation consisted, briefly, in dispersing the pigment particles in a binding medium and then covering a rag paper plaque with successive paint coats until its surface was completely hidden. For the preparation of the medium a sizing material was made by cooking parchment cuttings with water and then decanting the liquor, which was cooled to a gel and dried. The medium was subsequently prepared from one part by weight of the dried parchment size to ten parts of water. The pigment in each case was hand-ground. Medium was added, the quantity of which was determined by the absorption ratio of the pigment, such that a uniform working consistency was obtained for all samples. In this process a practical minimum of vehicle was used, a technique which gave a lean-vehicular paint structure; that is, the resultant film had a high pigment content and dried with a mat surface.

Listed in Table I will be found the pigments which were analyzed, as well as their origin and

TABLE I.

PIGMENT	ORIGIN	COMPOSITION
Red lead	Artificial	Lead oxide, Pb_3O_4
English vermilion	Artificial	Mercuric sulphide, HgS
Cadmium red	Artificial	Cadmium sulpho-selenide, CdS+CdSe
Madder lake	*	Organic†
Alizarin crimson	Artificial	Organic†
Carmine lake	*	Organic†
Dragon's blood	Natural	Organic
Realgar	Natural	Arsenic disulphide, As_2S_2
Venetian red	Artificial	Ferric oxide and calcium sulphate, $Fe_2O_3+CaSO_4$
Indian red	Artificial	Chiefly ferric oxide, Fe_2O_3, and some calcium sulphate, $CaSO_4$
Cadmium orange, medium	Artificial	Cadmium sulphide, CdS
Gamboge	Natural	Organic
Yellow ochre	Natural	Hydrous ferric oxide, $Fe_2O_3 \cdot nH_2O$, plus alumina and silica
Zinc yellow	Artificial	Zinc chromate, $ZnCrO_4$
Indian yellow	Natural	Organic
Saffron	Natural	Organic
Yellow lake (quercitron)	Natural	Organic†
Strontium lemon yellow	Artificial	Strontium chromate, $SrCrO_4$
Hansa yellow 5 G	Artificial	Organic†
Chrome yellow, medium	Artificial	Lead chromate, $PbCrO_4$
Cadmium yellow, light (cadmium lithopone)	Artificial	Cadmium sulphide, $CdS+BaSO_4$
Orpiment	Natural	Arsenic trisulphide, As_2S_3
Cobalt yellow	Artificial	Cobalt and potassium nitrites, $CoK_3(NO_2)_6 \cdot H_2O$
Raw sienna	Natural	Chiefly oxide and hydrous oxide of iron and aluminum plus silica
Burnt sienna	Natural	Chiefly oxides of iron and aluminum plus silica
Raw umber	Natural	Chiefly hydrous oxides of iron and manganese
Burnt umber	Natural	Chiefly oxides of iron and manganese
Malachite	Natural	Basic copper carbonate, $CuCO_3 \cdot Cu(OH)_2$
Chrome green, medium	Artificial	Chrome yellow mixed with Prussian blue
Verdigris	Artificial	Basic copper acetate, $Cu_3(OH)_2(C_2H_3O_2)_4$
Emerald green	Artificial	Copper aceto-arsenite, $Cu(C_2H_3O_2)_2 \cdot 3CuAs_2O_4$
Terre verte	Natural	Iron, potassium and magnesium silicates
Cobalt green	Artificial	Cobalt and zinc oxides, $CoO \cdot nZnO$
Viridian	Artificial	Hydrous chromic oxide, $Cr_2O_3 \cdot 2H_2O$
Cobalt blue	Artificial	Oxides of cobalt and aluminum, $CoO \cdot nAl_2O_3$
Genuine ultramarine	Natural	Sodium aluminum silicate with sulphur plus calcite, $Na_{8-10}Al_6Si_6O_{24} \cdot S_2 \pm + CaCO_3$

TABLE I (*Continued*).

PIGMENT	ORIGIN	COMPOSITION
Cerulean blue	Artificial	Oxides of cobalt and tin $CoO \cdot nSnO_2$
Azurite	Natural	Basic copper carbonate, $2CuCO_3 \cdot Cu(OH)_2$
Manganese violet	Artificial	Manganese phosphate
Cobalt violet	Artificial	Cobalt phosphate, $Co_3(PO_4)_2$
Smalt	Artificial	Silicate of cobalt and potassium
Blue verditer	Artificial	Basic copper carbonate, $2CuCO_3 \cdot Cu(OH)_2$
French ultramarine blue	Artificial	Sodium aluminum silicate with sulphur, $Na_{8-10}Al_6Si_6O_{24} \cdot S_2 \pm$
Indigo	*	Organic
Prussian blue	Artificial	Ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3$
Titanium white A	Artificial	Titanium oxide, TiO_2
Titanium white B	Artificial	75% barium sulphate, $BaSO_4$, and 25% titanium oxide, TiO_2
Zinc white	Artificial	Zinc oxide, ZnO
White lead	Artificial	Basic carbonate of lead, $2PbCO_3 \cdot Pb(OH)_2$
Lamp black	Artificial	Carbon, C
Ivory black	‡	Carbon plus calcium phosphate and carbonate, $C + Ca_3(PO_4)_2 + CaCO_3$

* This pigment comes from a natural source, but the process of preparation is so complex that it seems inaccurate to classify it as either natural or artificial.
† The base on which the dye is struck is an inorganic white inert.
‡ Most so-called ivory black sold in the artist trade today is bone black which is nearly identical in composition.

TABLE II.

PIGMENT	TRISTIMULUS VALUES			TRICHROMATIC COEFFICIENTS		
	X	*Y*	*Z*	*x*	*y*	*z*
Red lead	47.19	32.76	8.73	0.5321	0.3695	0.0984
English vermilion	34.96	22.26	10.04	0.5197	0.3309	0.1494
Cadmium red	32.84	20.78	7.47	0.5375	0.3402	0.1223
Madder lake	48.51	33.55	39.68	0.3985	0.2756	0.3259
Alizarin crimson	11.67	6.61	3.48	0.5361	0.3038	0.1601
Carmine lake	7.99	5.04	3.18	0.4929	0.3107	0.1964
Dragon's blood	6.83	4.94	3.54	0.4460	0.3228	0.2312
Realgar	42.17	32.35	12.11	0.4868	0.3734	0.1398
Venetian red	17.71	13.12	7.08	0.4672	0.3462	0.1866
Indian red	12.29	10.34	9.74	0.3797	0.3194	0.3009
Cadmium orange, medium	51.94	42.18	4.90	0.5245	0.4260	0.0495
Gamboge	34.14	29.87	4.89	0.4955	0.4335	0.0710
Yellow ochre	43.77	41.15	16.81	0.4303	0.4045	0.1652
Zinc yellow	78.05	82.57	13.37	0.4486	0.4746	0.0768
Indian yellow	43.40	40.01	5.13	0.4902	0.4519	0.0579
Saffron	34.40	33.45	9.28	0.4470	0.4345	0.1185
Yellow lake	28.49	27.24	8.87	0.4410	0.4217	0.1373
Strontium lemon yellow	74.10	84.36	19.80	0.4157	0.4732	0.1111
Hansa yellow 5 G	66.51	75.09	7.97	0.4447	0.5020	0.0533
Chrome yellow, medium	68.34	63.07	9.70	0.4843	0.4470	0.0687
Cadmium yellow, light	71.58	76.66	10.83	0.4500	0.4819	0.0681
Orpiment	65.55	64.45	16.40	0.4477	0.4402	0.1121
Cobalt yellow	48.53	50.48	9.50	0.4472	0.4652	0.0876
Raw sienna	22.33	20.03	7.19	0.4507	0.4043	0.1450
Burnt sienna	9.62	7.55	4.96	0.4347	0.3414	0.2239
Raw umber	7.66	7.44	5.05	0.3802	0.3693	0.2505
Burnt umber	5.74	5.08	3.53	0.4000	0.3540	0.2460
Malachite	34.97	41.77	42.84	0.2924	0.3493	0.3583
Chrome green, medium	11.40	16.04	8.94	0.3133	0.4410	0.2457
Verdigris	9.95	16.67	32.03	0.1696	0.2843	0.5461
Emerald green	22.71	39.12	31.00	0.2446	0.4215	0.3339
Terre verte	25.58	29.04	28.12	0.3092	0.3510	0.3398
Cobalt green	13.97	19.99	25.78	0.2339	0.3346	0.4315
Viridian	5.87	9.85	11.37	0.2167	0.3635	0.4198
Cobalt blue	18.42	16.81	67.22	0.1798	0.1641	0.6561
Genuine ultramarine	19.66	18.64	54.16	0.2126	0.2016	0.5858
Cerulean blue	16.76	18.19	51.85	0.1931	0.2096	0.5973
Azurite	9.51	9.26	27.34	0.2062	0.2008	0.5930
Manganese violet	31.76	26.99	44.60	0.3073	0.2612	0.4315

TABLE II (*Continued*).

PIGMENT	TRISTIMULUS VALUES			TRICHROMATIC COEFFICIENTS		
	X	*Y*	*Z*	*x*	*y*	*z*
Cobalt violet	14.44	9.34	27.49	0.2817	0.1821	0.5362
Smalt	11.99	8.25	42.93	0.1898	0.1306	0.6796
Blue verditer	16.62	16.46	54.46	0.1899	0.1880	0.6221
French ultramarine blue	11.90	7.84	48.40	0.1747	0.1151	0.7102
Indigo	4.22	3.62	5.43	0.3180	0.2728	0.4092
Prussian blue	1.52	1.30	2.47	0.2883	0.2453	0.4664
Titanium white A	93.50	95.75	110.02	0.3124	0.3199	0.3677
Titanium white B	93.45	95.66	110.17	0.3122	0.3196	0.3682
Zinc white	92.55	94.88	109.03	0.3122	0.3200	0.3678
White lead	85.54	87.32	96.47	0.3176	0.3242	0.3582
Lamp black	3.98	4.08	4.86	0.3081	0.3157	0.3762
Ivory black	2.14	2.22	2.63	0.3055	0.3176	0.3769

chemical composition insofar as it is known. Many of the pigments can be obtained in both the natural and the artificial form so that designation in the table refers in such cases to the actual pigment used.

The analysis was made through the use of a recording photoelectric spectrophotometer.[1, 2] The resultant spectrophotometric curves have been placed in interrelated groups and are shown in Figs. 1 to 4. The tristimulus values and trichromatic coefficients have been computed using I.C.I. Illuminant C, standard observer, and coordinate system and are to be found in Table II, while Table III includes the corresponding values of the dominant wave-length, relative brightness, and excitation purity of the samples measured.

In order to give the reader an indication of

TABLE III.

PIGMENT	DOMINANT WAVE-LENGTH (MILLIMICRONS)	RELATIVE BRIGHTNESS (PERCENT)	EXCITATION PURITY (PERCENT)
Red lead	596.6	32.8	73.8
English vermilion	608.1	22.3	59.9
Cadmium red	604.8	20.8	67.3
Madder lake	496.5c	33.6	32.9
Alizarin crimson	628.0	6.6	57.2
Carmine lake	622.0	5.0	47.4
Dragon's blood	610.2	4.9	38.1
Realgar	593.0	32.4	62.6
Venetian red	599.2	13.1	50.2
Indian red	610.5	10.3	19.5
Cadmium orange, medium	586.9	42.2	86.9
Gamboge	584.0	29.9	81.2
Yellow ochre	581.9	41.2	55.9
Zinc yellow	575.8	82.6	79.7
Indian yellow	581.6	40.0	84.8
Saffron	579.6	33.4	68.4
Yellow lake	580.6	27.2	63.4
Strontium lemon yellow	572.3	84.4	70.4
Hansa yellow 5 G	573.2	75.1	85.9
Chrome yellow, medium	581.6	63.1	81.8
Cadmium yellow, light	575.3	76.7	81.9
Orpiment	579.1	64.4	70.2
Cobalt yellow	576.5	50.5	76.8
Raw sienna	584.2	20.0	61.3
Burnt sienna	598.5	7.6	40.1
Raw umber	581.5	7.4	33.0
Burnt umber	589.5	5.1	34.2
Malachite	515.2	41.8	6.6
Chrome green, medium	552.4	16.0	34.7
Verdigris	488.2	16.7	54.8
Emerald green	511.9	39.1	22.8
Terre verte	549.8	29.0	9.2
Cobalt green	493.8	20.0	27.0
Viridian	497.1	9.8	31.9
Cobalt blue	474.6	16.8	65.5
Genuine ultramarine	474.4	18.6	49.2
Cerulean blue	478.9	18.2	54.5
Azurite	475.6	9.3	51.3
Manganese violet	553.7c	27.0	21.6
Cobalt violet	560.3c	9.3	48.5
Smalt	466.2	8.2	68.1
Blue verditer	476.4	16.5	58.7
French ultramarine blue	467.8	7.8	75.1
Indigo	540.0c	3.6	19.0
Prussian blue	563.5c	1.3	24.4
Titanium white A	565.5	95.8	1.5
Titanium white B	570.0	95.7	1.4
Zinc white	569.5	94.9	1.5
White lead	576.7	87.3	4.1
Lamp black	487.5	4.1	0.8
Ivory black	494.5	2.2	1.7

TABLE IV.

PIGMENT	DOMINANT WAVE-LENGTH (MILLIMICRONS)	RELATIVE BRIGHTNESS (PERCENT)	EXCITATION PURITY (PERCENT)
Emerald green			
Sample 1	511.9	39.1	22.8
Sample 2	509.2	39.1	23.2
Sample 3	509.2	38.8	22.9
Cerulean blue			
Sample 1	478.8	18.2	54.5
Sample 2	478.7	18.6	55.0
Sample 3	478.7	18.1	54.9
Venetian red			
Sample 1	599.2	13.1	50.2
Sample 2	599.0	12.9	51.6
Sample 3	599.0	13.2	50.7

[1] A. C. Hardy, "A New Recording Spectrophotometer," J. Opt. Soc. Am. **25**, 305 (1935).

[2] The samples were placed over a two-inch diameter hole in an eight-inch diameter integrating sphere which was coated on the inside with magnesium oxide. The actual area measured was approximately circular and had a diameter of about one inch. The standard comparison white was a freshly prepared magnesium oxide surface, a surface having a substantially nonselective reflectance of 97.4 percent, according to J. S. Preston, "The Reflection Factor of Magnesium Oxide," Trans. Opt. Soc. **31**, 15 (1929–30).

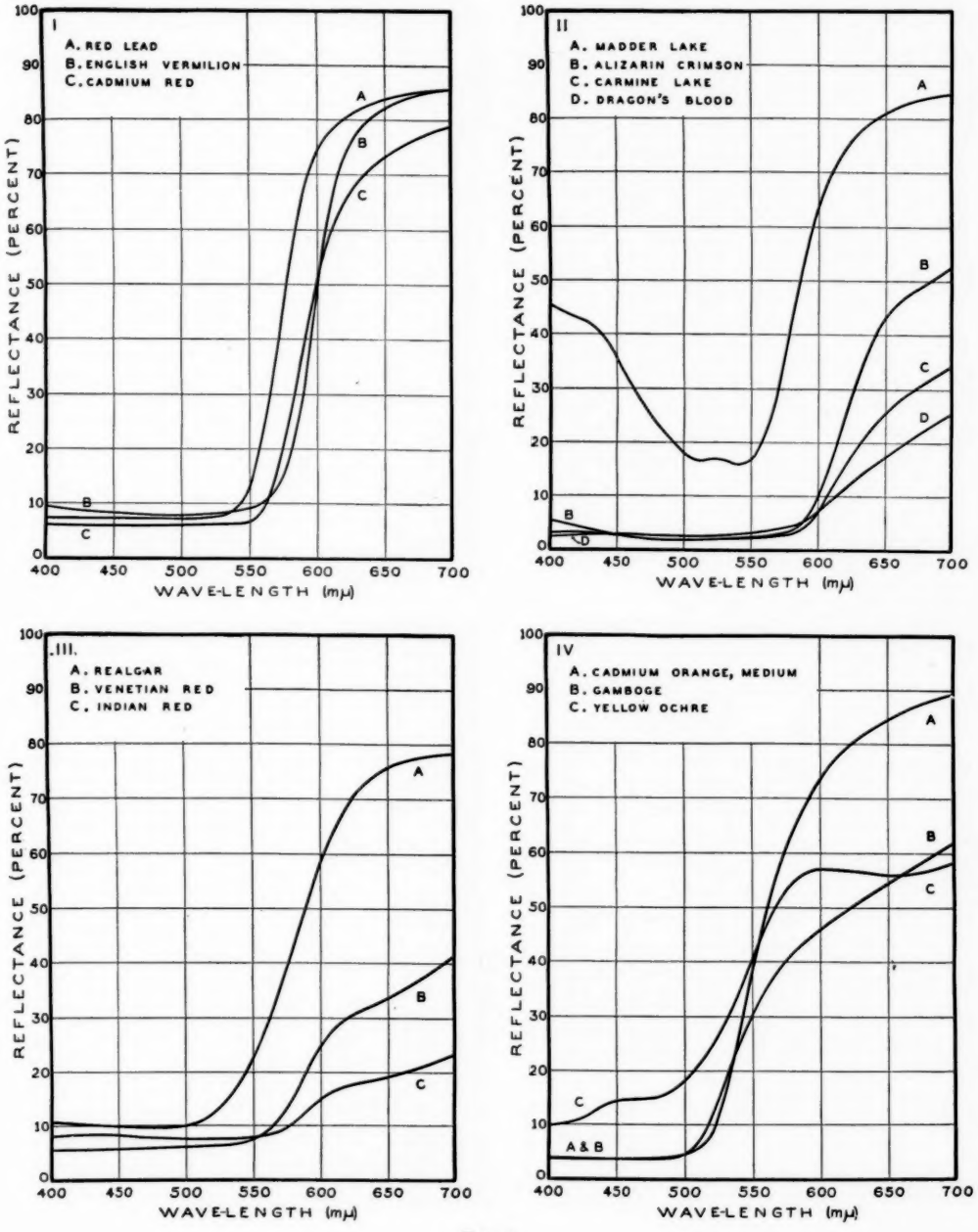
I
A. RED LEAD
B. ENGLISH VERMILION
C. CADMIUM RED
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
II
A. MADDER LAKE
B. ALIZARIN CRIMSON
C. CARMINE LAKE
D. DRAGON'S BLOOD
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
III.
A. REALGAR
B. VENETIAN RED
C. INDIAN RED
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
IV
A. CADMIUM ORANGE, MEDIUM
B. GAMBOGE
C. YELLOW OCHRE
A & B
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)

FIG. 1.

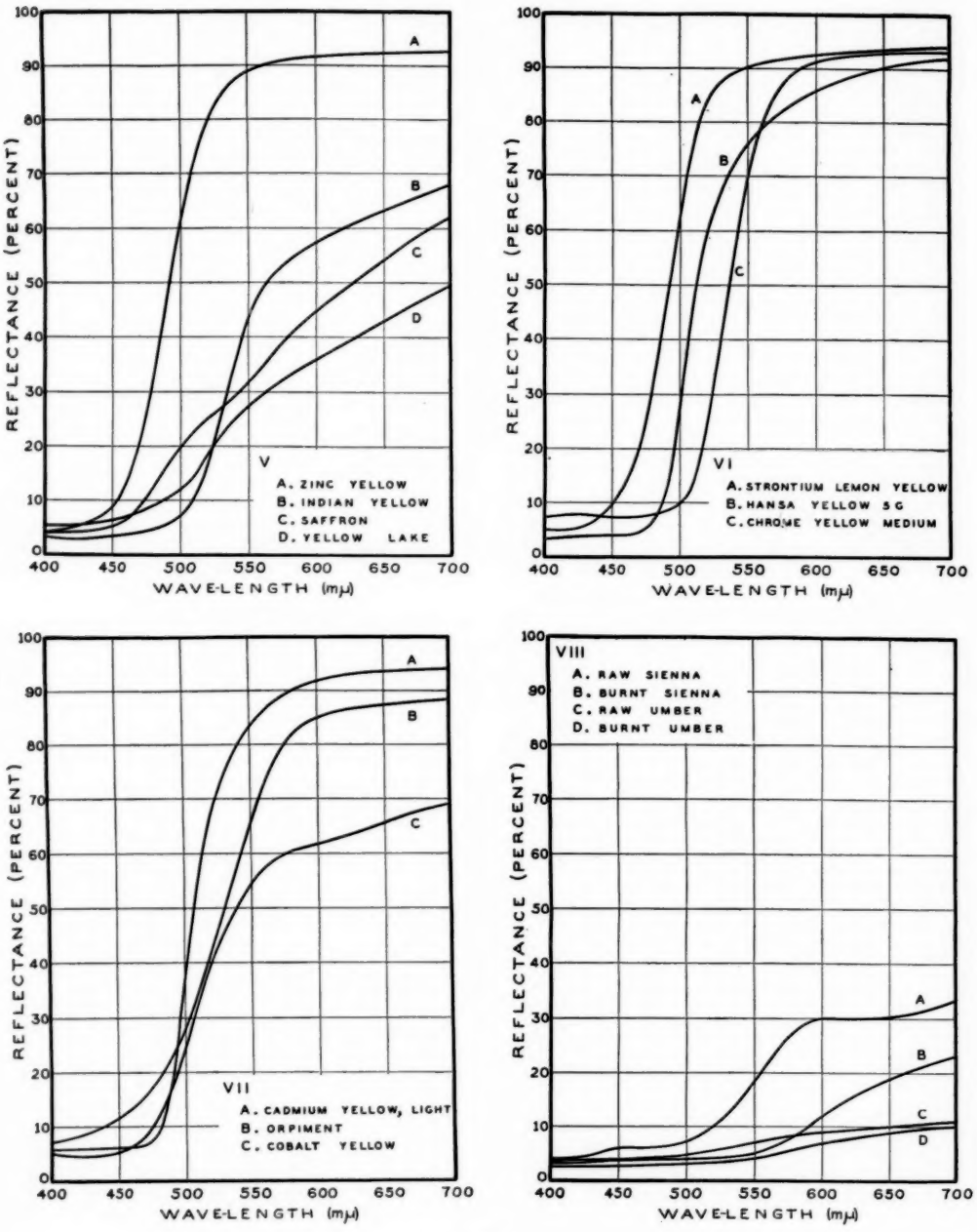
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
V
A. ZINC YELLOW
B. INDIAN YELLOW
C. SAFFRON
D. YELLOW LAKE
VI
A. STRONTIUM LEMON YELLOW
B. HANSA YELLOW 5G
C. CHROME YELLOW MEDIUM
VII
A. CADMIUM YELLOW, LIGHT
B. ORPIMENT
C. COBALT YELLOW
VIII
A. RAW SIENNA
B. BURNT SIENNA
C. RAW UMBER
D. BURNT UMBER

FIG. 2.

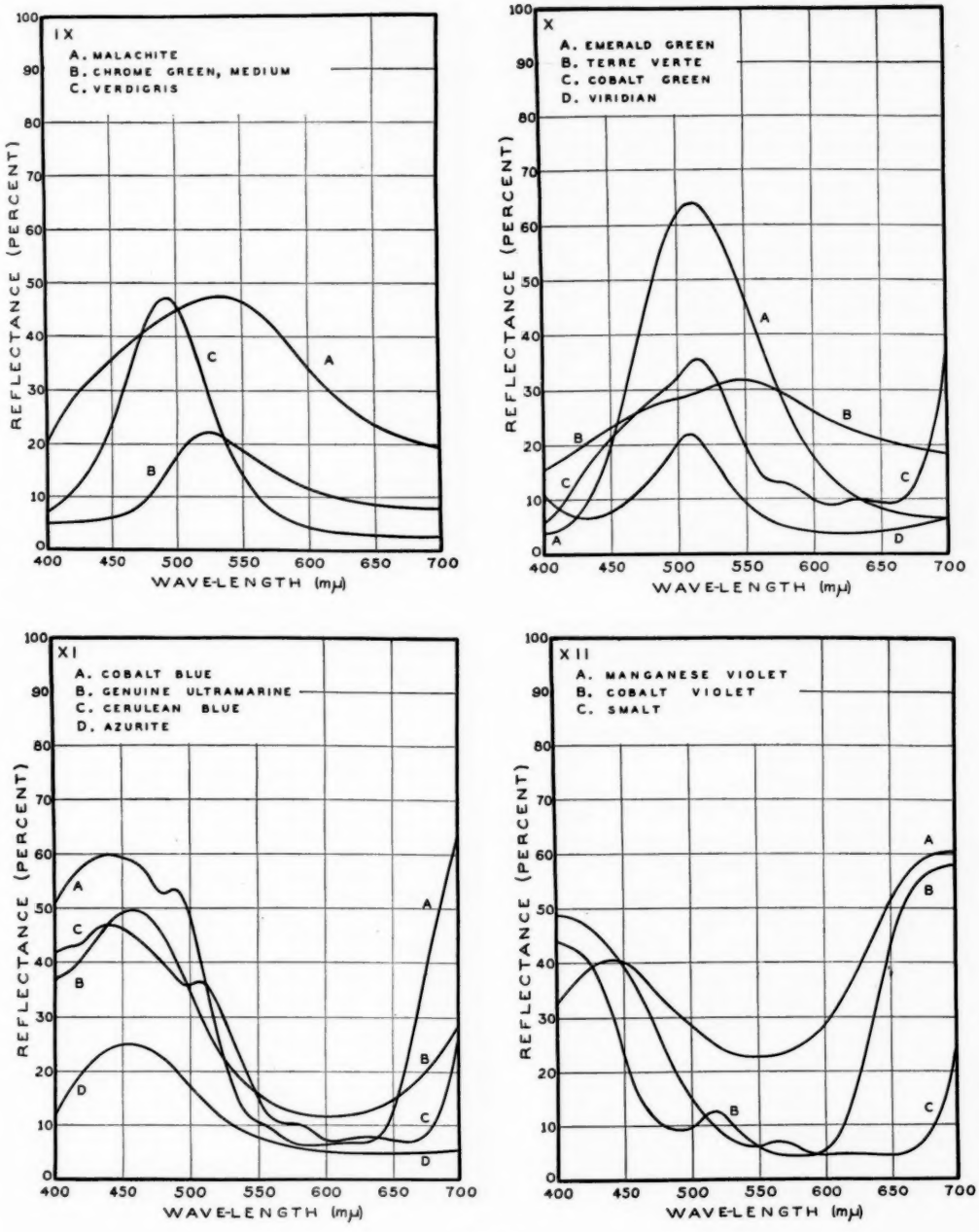
IX
A. MALACHITE
B. CHROME GREEN, MEDIUM
C. VERDIGRIS
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
X
A. EMERALD GREEN
B. TERRE VERTE
C. COBALT GREEN
D. VIRIDIAN
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
XI
A. COBALT BLUE
B. GENUINE ULTRAMARINE
C. CERULEAN BLUE
D. AZURITE
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)
XII
A. MANGANESE VIOLET
B. COBALT VIOLET
C. SMALT
REFLECTANCE (PERCENT)
WAVE-LENGTH (mμ)

FIG. 3.

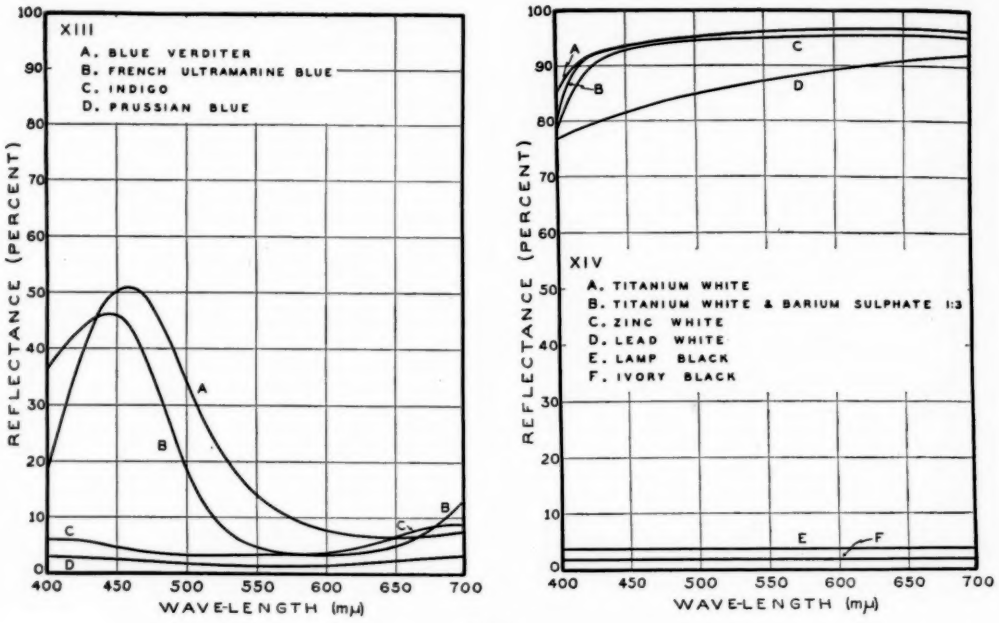

FIG. 4.

the precision of the results from the standpoint of irregularities in the experimental technique, a table of comparative dominant wave-length, brightness and purity values has been prepared for three samples of each of three pigments picked at random. For each group the samples were prepared using the same paint mixture and employing the technique described earlier. The final results of this comparison are given in Table IV.

The author wishes to express his sincere appreciation to Professor Arthur C. Hardy and to Mr. S. Q. Duntley for their helpful suggestion and to Mr. J. J. Hanlon for the integration of the curves. Special acknowledgment is given to the research staff of the Fogg Art Museum for preparing the samples for measurement, especially to Mr. George L. Stout for his helpful guidance in assisting this work.

Measuring Visibility Under Preventable Glare

MATTHEW LUCKIESH AND L. L. HOLLADAY
Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland
(Received March 2, 1939)

THE visibility of the object or task and the comfort of the observer are two aspects of every situation under which seeing[1] is done. All brightnesses in the visual field, regardless of their magnitude, area and location, affect visibility and comfort obviously or unobviously. Therefore, there can be no actual condition which is entirely glareless. The ideal objective is to produce seeing conditions under which the effects of glare are minimized. As commonly used, the term *glare* generally means unnecessary or *preventable* glare from obviously high brightnesses prominent in the visual field.

The reduction of the visibility of objects due to glare is apparently of the same character and degree that would result from a certain amount of illuminated fog. In other words, any glare condition can be expressed as a veiling brightness which would reduce visibility equivalently. Holladay[2] found that, for a single glare-source located at an angle D to the line of vision, the equivalent veiling brightness which would cause the same reduction in visibility as the glare-source is (1) directly proportional to the illumination E in footcandles due to the glare-source at the eyes and in a plane normal to the line connecting the eyes and the glare-source; and (2) inversely proportional to the angle D in degrees raised to a power N varying from 1.5 to 2.0. The equivalent veiling brightness in footlamberts may be empirically expressed as

$$B = 23E/D^{1.8}.$$

Holladay also found with two or more glare-sources the resultant veiling brightness is approximately the sum of the equivalent veiling brightnesses due to each acting alone.

For years we have considered making a physical transparent screen having the proper form and diffusing characteristic so that its *veiling brightness* due to glare from one or more glare-sources, regardless of their location in the visual field, would be properly related to the reduction of visibility actually caused by glare-sources. Obviously either the brightness of such a screen could be calibrated against visibility reduction or it could be properly placed before a Luckiesh-Moss visibility meter[3] with assurance that it would affect the readings of visibility in proportion to the disability glare if present. The L-M visibility meter is calibrated for a field of uniform brightness and should be shielded from prominent sources of preventable glare excepting when only relative data are desired. It can be used without this precaution in most situations arising in practice. In its laboratory uses it is easy to standardize the appropriate conditions.

Obviously an appropriate diffusing screen must acquire veiling brightness which is appropriate for the location of the glare-source in the visual field as well as for the footcandles at the eyes of the observer due to the glare-source. More than a hundred screens were studied in preliminary experiments. Various liquids, either inherently slightly diffusing or containing materials purposely added, were experimented with. Various varnishes with and without additional media were tried. Delicately etched glasses of different forms were studied with the hope of obtaining a perfect medium. Finally it appeared that photographic films properly exposed, developed, treated with varnish and combined in various ways were most generally promising because they are transparent and also possess other necessary and desirable characteristics. For the purpose of illustration we have chosen two of the simpler ones as being suitable enough in acquiring appropriate veiling brightnesses under the entire range of glare conditions. How closely these two transparent diffusers approach the computed ideal is shown in Fig. 1. The actual data are presented in Table I for various angles of a given glare-source with the line of vision which is

[1] Matthew Luckiesh and Frank K. Moss, *The Science of Seeing* (D. Van Nostrand Company, New York, 1937).
[2] L. L. Holladay, "The Fundamentals of Glare and Visibility," J. Opt. Soc. Am. **12**, 271 (1926).
[3] M. Luckiesh and Frank K. Moss, "Visibility—Its Measurement and Significance in Seeing," J. Frank. Inst. **220**, 431 (1935).

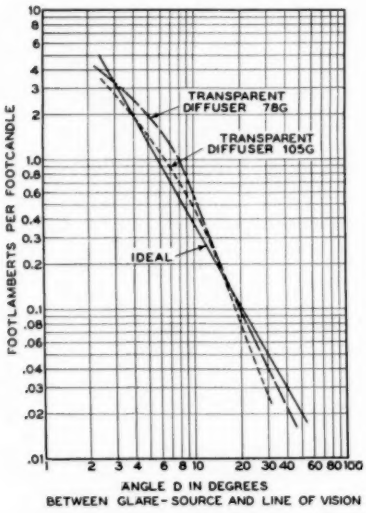

FIG. 1. Veiling brightnesses of an ideal and two typical transparent diffusers for various angles D of a glare-source to the line of vision.

normal to the plane of the transparent diffuser in its simplest form.

The transparent diffuser 105G consisted of two glass lantern-slide plates, slightly exposed and normally developed, with the emulsion sides placed face to face. Transparent diffuser 78G consisted of a single lantern-slide plate slightly exposed and developed but with the emulsion coated with a varnish. The measurements of the veiling brightness produced by a given glare-source at various angles with the line of vision were made with a Luckiesh-Taylor brightness meter.[4]

The brightnesses of more than a hundred diffusing receivers were investigated under definite conditions of preventable glare. A tube was used in tandem with the brightness meter and the diffusing receiver could be easily placed at the outer end of the tube and could be as readily removed. By using a black background the veiling brightness of the diffusing receiver could be measured. The ratio of the veiling brightness to the brightness of the background of the test-object is an important factor in any lighting and seeing condition. Obviously a device for appraising the reduction in visibility due to one or more glare-sources need consist only of a suitably sensitive brightness meter and an appropriate diffusing receiver which for this use does not need to be transparent. Of course, the veiling brightness of the diffusing receiver must be measured by viewing it against a perfectly dark background, or the brightness of a given test-object must be measured with and without the added veiling brightness of the receiver. By subtraction in the latter case the veiling brightness due to the glare-source (or sources) is obtained. We also found that the ratio of the veiling brightness to the brightness of the test-object or its background is significant and useful. Helpful data were also furnished for the development of a proper transparent diffuser or diffusing receiver by measurements of transmission-factors of the screens and their veiling-brightness characteristic. These and other details of the development of these screens are omitted.

Figure 1 and Table I supply data relating to the use of a proper transparent diffuser and its veiling brightness as an indicator of the reduction of visibility due to any combination and location of glare-sources. Fig. 2 shows the application of it to the Luckiesh-Moss visibility meter. The latter is attached to the left end of the binocular device shown. The diffusing receiver, which must be transparent for this usage, is placed at the front position F as a receiver of the visibility-reducing light from the glare-sources. Actual measure-

TABLE I. *The brightnesses in footlamberts per footcandle of two transparent diffusers (compared with the ideal) due to illumination from a glare-source at various angles D with the line of vision. The glare illumination was measured on a plane normal to the beam. The angle D indicates the position of the glare-source in respect to the line of vision and also to the normal to the plane of the transparent diffuser.*

	BRIGHTNESS OF TRANSPARENT DIFFUSER IN FOOTLAMBERTS PER FOOTCANDLE		
ANGLE D IN DEGREES	IDEAL	105G	78G
3	3.18	2.69	3.22
5	1.27	1.42	1.53
10	0.364	0.49	0.56
15	0.176	0.174	0.183
20	0.105	0.083	0.096
25	0.070	0.045	....
30	0.050	....	0.040
40	0.030	....	0.022
Transmission factor	....	0.79	0.83

[4] Matthew Luckiesh and A. H. Taylor, "A Brightness Meter," J. Opt. Soc. Am. **27**, 132 (1937).

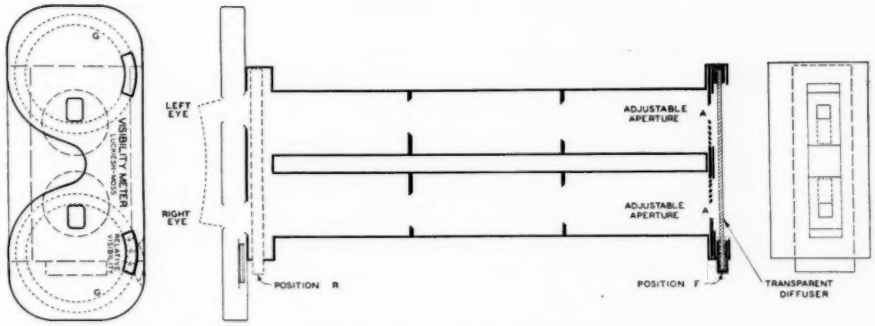

FIG. 2. A sketch of a device consisting of a transparent diffuser for use in combination with the Luckiesh-Moss visibility meter for properly measuring visibility under any condition of glare.

ments of visibility of objects can thus be made with the eyes screened from the glare-sources, and, therefore, in a suitable state of adaptation. The transparent diffuser is next placed at the rear position *R* where it as well as the eyes are screened from the glare-sources. Measurements of visibility of objects now are equivalent to those if no glare-sources were present. Obviously if the transparent diffuser possesses suitable optical characteristics as indicated in Fig. 1 and Table I, the reduction in visibility due to glare is determined by such a procedure involving the devices shown in Fig. 2.

In order to compare the results of this device with those obtained with the naked eye, tests were made with the same test-objects of various sizes and contrasts used by Cobb and Moss[5] in obtaining their extensive data on visibility with the naked eye with and without certain specified glare conditions. The results of this comparison for both the binocular type shown in Fig. 2 and a monocular type (which was approximately the same as one-half of that shown) agree within a few percent with the results obtained by Cobb and Moss with the naked eyes. Thus it is possible to measure quickly and easily the effects of glare upon visibility of objects. Without such a combination of devices measurements are so difficult and time-consuming as to be generally impossible in practice.

[5] P. W. Cobb and F. K. Moss, "Glare and the Four Fundamental Factors in Vision," Trans. I. E. S. **23**, 1104 (1928).

A Pocket-Size Ultraviolet Meter

A. H. TAYLOR
Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland
(Received March 8, 1939)

A PRACTICABLE method of measuring ultraviolet radiant energy involves the use of a photo-tube properly sensitive to the spectral region where measurements are desired. About three years ago Dr. L. R. Koller and the present author described[1] a photo-tube developed to measure that portion of the ultraviolet spectrum which is effective in producing erythema, tan and antirachitic action. This tube employs an alloy of cadmium and magnesium as the cathode.

The current obtainable from a photo-tube is generally too small to measure directly unless the intensity of radiation is high or a very sensitive galvanometer is used. Hence some type of amplifier is necessary. The present author has previously described[2] some meters employing a vacuum tube amplifier and cadmium-alloy photo-tube. One of these, an indicating type, gives instantaneous measurements of intensity of ultraviolet radiation. It has a wide sensitivity range, the most sensitive condition being about 5000 times as sensitive as the least sensitive condition. The meter has proved to be a very valuable aid to research and could be equally valuable in many applications of ultraviolet energy where the absence of meaningful measurements is a deterrent to progress.

A simpler meter, suitable for use in many fields of research, has recently been developed in this laboratory. It employs the same type of photo-tube but does not require any indicating meters or batteries. Fig. 1 is a photograph of the meter and Fig. 2 shows the wiring diagram. The complete meter is contained in a case $3\times3\frac{1}{2}\times7\frac{1}{4}$ inches and obviously is a very convenient pocket-size instrument.

The meter connects directly to the 110–125-volt a.c. mains by means of an extension cord plugged into the end. Referring to Fig. 2, R is a small, high voltage copper oxide rectifier which gives half-wave rectification and charges condenser C to furnish direct current for the photo-tube PT. Switch S is used to connect the photo-tube in series with any one of three condensers of different capacities, thus extending the range and convenience of the instrument. A $\frac{1}{4}$-watt neon lamp N is connected in series with a telephone receiver T across the terminals of the condenser selected by S.

FIG. 1. Photograph of complete pocket-size ultraviolet meter.

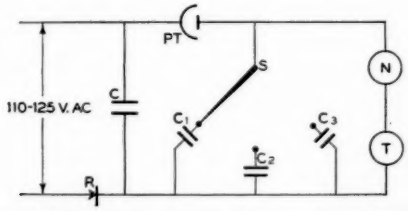

FIG. 2. Wiring diagram of meter (see text for explanation).

When the photo-tube is exposed to ultraviolet radiant energy, current flows in the photo-tube and charges the condenser connected to switch S. When the voltage across this condenser reaches a value of approximately 75 volts, a momentary discharge takes place through the neon lamp and telephone receiver, causing an audible click in the receiver. The intensity of the ultraviolet energy is directly proportional to the rate of the clicks. By use of three condensers differing in capacity

[1] L. R. Koller and A. H. Taylor, "Cadmium-Magnesium Alloy Photo-Tubes," J. Opt. Soc. Am. **25**, 184 (1935).
[2] A. H. Taylor, "Portable Ultraviolet Meters," J. Opt. Soc. Am. **24**, 183 (1934).

the sensitivity may be varied over a range of about 100 to 1.

The photo-tube used in the meter is of the vacuum type. Since the applied voltage is always well above the approximate saturation value, the meter rate is affected very little by line voltage variations. By trial it was found that 30 percent increase in the applied voltage increased the rate of clicking only about 4 percent.

Obviously, the spectral sensitivity of the meter

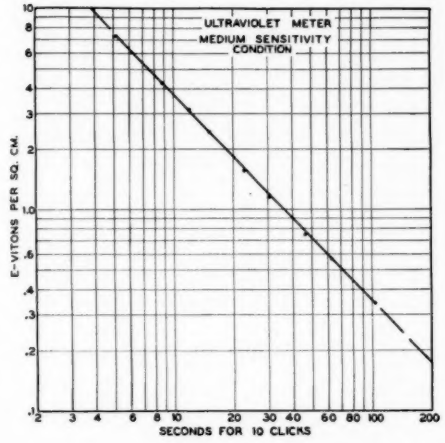

FIG. 3. Typical calibration curve.

is dependent upon the spectral sensitivity of the photo-tube used and of any filters which might be superposed. The cadmium-alloy photo-tube referred to above is insensitive to radiation longer than about λ3300 angstroms, and its short wavelength limit is dependent upon the transmission of the bulb. In this case Corex glass is used but other glasses and even quartz may be used if the spectral range is to be altered. Experience has shown that it appraises artificial sources approximately in proportion to their erythemal power when the Corex bulb is used, but that sunlight is somewhat overrated.

Figure 3 shows the calibration for one condition for one of these meters. Similar calibrations, showing the exposure required to produce a minimum perceptible erythema or reddening of the skin, can also be readily derived.

FIG. 4. A battery-operated meter of the same type as Fig. 1, containing three 45-volt batteries.

Others[3, 4] have previously described the use of a photo-tube with condensers, batteries and neon lamp to measure radiant energy. However, the essential novelty of the meter described above is the elimination of batteries by use of a copper oxide rectifier and the use of a telephone receiver to make the neon lamp flashes audible.

We have recently developed another meter of the same type to operate from commercially available "midget" dry batteries. While not as compact as the meter described above, it weighs only $5\frac{1}{2}$ pounds complete and is in a case $4\frac{3}{4} \times 7 \times 7$ inches. Since it is entirely independent of any outside source of power, it has many obvious advantages. Fig. 4 is a photograph of one of these meters containing three 45-volt "midget" batteries.

In order to make this type of meter work satisfactorily in all types of weather, especial care must be taken to avoid leakages due to moisture.

[3] S. O. Pearson and H. St. G. Anson, Proc. Phys. Soc. **34**, 204 (1922).
[4] J. H. J. Poole, Sci. Proc. Roy. Soc. Dublin **19**, 17 (1928).

Illuminator for Visual Spectrum Line Densitometers*

MORRIS SLAVIN†
Bureau of Mines, Eastern Experiment Station, College Park, Maryland
(Received March 6, 1939)

THIS laboratory, for the past three years, has employed a Gaertner densitometer in the measurement of line densities for spectrochemical analysis. This instrument, after two minor changes had been made, has proved so satisfactory for this purpose that a note describing the changes should be of interest to other workers in the field.

The densitometer is of the visual type,[1] based on the original design of Hartmann. A beam of light is split into two equal portions, one passing through the photographic image, the other through a neutral density wedge capable of adjustment. The two beams are recombined by a Lummer-Brodhun cube, the measurement being made by varying the position of the wedge until the intensity of its beam matches the beam passing through the spectrum line.

The illuminant supplied with the instrument was an ordinary 75-watt incandescent lamp. This proved inadequate for two reasons. The intensity was uncomfortably low, even when working at medium densities. The second and more serious fault was the poor color match between the beam from the wedge and the beam from the spectrum line, the former having a bluish cast and the latter a brownish one. When the lamp was changed to a type H-3 mercury vapor lamp (made by the General Electric Company) both difficulties disappeared.

The second modification in the densitometer consisted in replacing the wedge with which the instrument is ordinarily equipped, having a range of zero to 4.0, with one having a range of zero to 3.0. This latter range matches more closely the usable density scale of the slow plates customarily employed in spectrochemical work. Thus the whole of the wedge scale is in use, whereas the densest quarter of the former wedge had never been used.

The subjective type of densitometer has the reputation of being old-fashioned, compared to the modern photoelectric instrument, but after the changes described here had been made it was found that it had several noteworthy advantages over the more popular type. Readings generally check to about 0.01 density, which compares favorably with photoelectric types, but this precision is maintained over the whole of the density range, whereas photoelectric densitometers become ineffective at higher densities. In practice this means that with the latter instrument only the lower portion of the characteristic curve is used and thus a part of the latitude of the plate is wasted. In the literature this is described as a "preference" to work near the toe of the curve. In addition, the visual instrument is compact, much more rugged, cheaper, and may be calibrated to read directly in units of density. A further advantage is that blemishes in the photographic image, such as pinholes or dustlines, can be disregarded when making the match, whereas with photoelectric densitometers they may cause errors.

* Published by permission of the Director, Bureau of Mines, U. S. Department of the Interior. (Not subject to copyright.)
† Assistant Metallurgist, Nonmetals Division.
[1] S. Jacobsohn and W. H. Kliever, J. O. S. A. **25**, 244 (1935).

An Improved 365-mμ Mercury Monochromat

ELIZABETH M. STAATS
Fish-Schurman Corporation, New York, New York
(Received March 10, 1939)

SUBSEQUENT to the designing of the monochromatic filters for isolating mercury lines, previously described,[1] a number of new optical filter glasses were made available by the Jena Glass Works.[2] Among these are several which have high transmissions in the ultraviolet region, and the series of filters designated WG, which have fairly sharp cut-offs at successively shorter wave-lengths ranging from 375 mμ to 200 mμ, are of especial interest.

The effectiveness and suitability of a filter to be used as a component in a two-filter combination is largely determined by its transmission at the wave-length for which the filter is designed and by its opacity at adjoining wave-lengths. The new 365-mμ monochromat has two components and was designed in accordance with the method outlined in the previous paper.[3] A comparison of the ratios of the transmissions at the wave-lengths 365 mμ and 334 mμ for the WG-2 and BG-10 filters showed the former to be far superior for eliminating radiant energy at 334 mμ. Therefore, the Jena WG-2 filter was selected for use in combination with Jena UG-2, replacing the BG-10 component of the original design. The UG-2 filter transmits 365 mμ freely and eliminates the longer wave-lengths.

In order to obtain maximum effectiveness in the design of a monochromatic filter the spectral energy distribution of the source and the spectral response of the receiver must be included in the computations. The energy distribution of the type H-3 mercury lamp was used. Two filters, Nos. 3 and 4, replacing Nos. 1 and 2 of the previous group, were designed. Filter No. 3 was designed for use with either a thermopile, a G. E. type PJ-22 (or PJ-23) caesium surface photo-tube or the Weston Photronic barrier-layer cell. Filter No. 4 was designed for use with the G. E. type FJ-76 sodium-in-quartz photo-tube.

[1] Elizabeth M. Staats, J. Opt. Soc. Am. **28**, 112 (1938).
[2] See supplementary catalog 5990, "Jenaer Farb- und Lichtfilterglaeser."
[3] Reference 1, pp. 115 and 116.

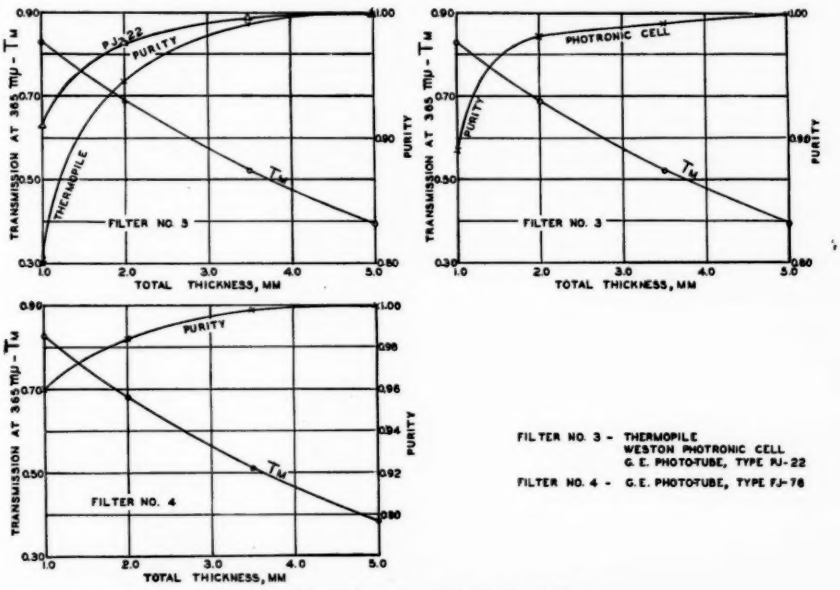

FIG. 1. Monochromats designed for 365 mμ.

TABLE I. *Monochromats for type H-3 mercury lamp.*

DESIGNED WAVE-LENGTH, mμ	FILTER NO.	RECEIVER	FILTER COMPONENTS		THICKNESS RATIO, *R* (I/II)	AUXILIARY FILTER
			I	II		
365	3	Thermopile	Jena WG-2, Melt No. 25051	Jena UG-2, Melt No. 3017	0.4	Water cell, 1 cm
		Weston Photronic cell	Jena WG-2, Melt No. 25051	Jena UG-2, Melt No. 3017	0.4	—
		G. E. photo-tube, type PJ-22	Jena WG-2, Melt No. 25051	Jena UG-2, Melt No. 3017	0.4	—
	4	G. E. photo-tube, type FJ-76	Jena WG-2, Melt No. 25051	Jena UG-2, Melt No. 3017	0.75	—

In this design the best ratio between the thicknesses of the components was selected, depending on the spectral sensitivity of the receivers. The best choice of the thickness of the combination can then be made as a compromise of those conflicting characteristics, high light transmission and spectral purity. This choice will naturally vary with the application in which the filter is to be used. See Table I.

Curves showing T_M (transmission at the wavelength for which the monochromat was designed) and the purity of the energy transmitted are given in Fig. 1. From these curves appropriate thicknesses for the components can be selected for various applications. In any specific problem demanding very precise data, filters of this design could be computed for the particular lamp, receiver and filters employed. The data given above will, however, be adequate for the vast majority of applications.